ABSTRACT FOR THE 1ST INTERNATIONAL WORKSHOP ON HIGH-ORDER CFD METHODS

Sreenivas Varadan[a], Kentaro Hara[b], Eric Johnsen[a], Bram Van Leer[b]

a. Department of Mechanical Engineering, University of Michigan, Ann Arbor

b. Department of Aerospace Engineering, University of Michigan, Ann Arbor

TEST CASE C1.5

1. Code Description

- Discretization:

The code used to numerically simulate this test case is based on the Discontinuous Galerkin (DG) method.

- Relevant solvers:

For DG, we use up to P4 elements for the sub-cell solution representation. Explicit fourth order Runge-Kutta time (RK4) and fifth order Runge-Kutta (RK5) stepping is used.

- High order capability:

While the DG code can be extended to arbitrarily high order, we have restricted ourselves to using up to P4 elements for the DG method.

- Parallel Capability:

Our codes are parallelized using a 3-D Domain decomposition based on the message-passing interface (MPI).

- Post Processing:

We use the Hdf5 Parallel libraries for Parallel I/O. Hdf5 is a commonly used scientific data format for large datasets. The Visualization is done with the Visit Visualization software.

2. Case Summary

- Convergence criteria:

This problem is not marched up to steady state. Rather, the errors in entropy are used to establish the order of convergence.

- Machines used:

160 processors at the Center for Advanced Computing cluster at the University of Michigan were used for all the cases.

- Taubench CPU times:

On one core of the Nyx machine, one TauBench unit is equivalent to 16.5 seconds of compute time. Here, we define one work unit as -:

1 WU = Computing time (T2) / Taubench benchmark time (T1).

3. Meshes

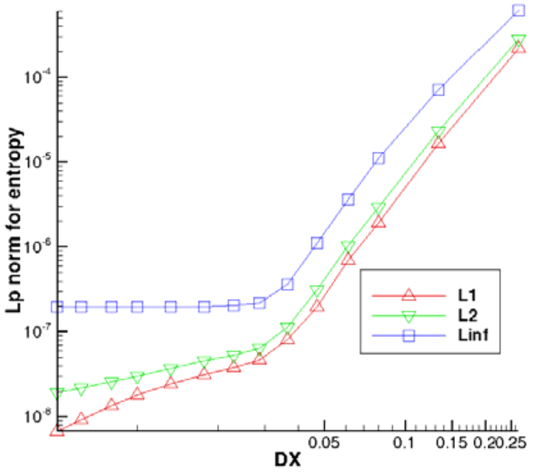
For this test case, we have used structured meshes with rectangular elements. This is expected to optimize the order of accuracy for this smooth test case.

4. Results

1-D

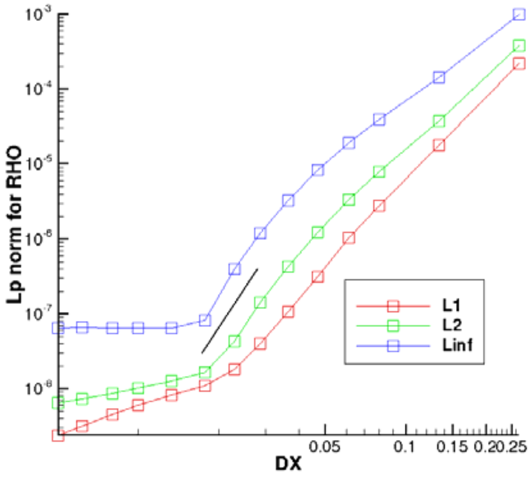
Here we show that in 1-D the exact solution can be used to find out the errors in primitive variables (density, velocity, pressure) and hence find the order of accuracy rather than by using the entropy errors. For this test case, when the ratio of specific heats is 3, the characteristics become straight and hence the backward method of characteristics can easily be used to find the exact solution. In this case we are sure the solution remains infinitely differentiable for all times.

Fig.1:



L1, L2, Linf Entropy errors vs. dx for P = 2

Fig.2:



L1, L2, Linf Entropy errors vs. dx for P = 3

(Black line indicates the expected slope)

Below is a comparison of entropy errors with errors in primitive variables when the exact solution is available. A 1-D P2, P3 DG code is used to make this comparison. The expected spatial order of accuracy for this smooth (infinitely differentiable) problem in 1-D is (2P+1), i.e., 5 for P2 and 7 for P3 elements, and this level of accuracy may very well be achievable in 2 and 3 dimensions. When making a discretization for a given value of P one therefore has to make sure that all its building blocks are consistent with such a high order, notably the Gaussian quadrature routine used.

In a time-dependent calculation this level of accuracy can be maintained only if the temporal integration is accurate enough. It may be necessary to use a time-marching scheme of order 2P+1, which RK5 falls short of for P3.

Alternatively, the time marching should be done with sufficiently small time steps.

Comparison of errors in primitive variables vs. Entropy errors in 1-D

Table.1: P=2 Primitive variables error vs. entropy error

# of grids	Density	(rate)	Velocity	(rate)	Pressure	(rate)	Entropy	(rate)
30	2.53E-003		8.18E-004		2.02E-004		3.93E-03	
60	3.67E-004	2.7847	2.17E-004	1.9123	3.45E-005	2.5519	3.11E-04	3.6604
100	8.34E-005	2.9011	6.23E-005	2.4459	8.59E-006	2.7233	3.71E-05	4.1603
130	3.96E-005	2.8356	3.07E-005	2.6999	4.23E-006	2.6983	1.17E-05	**4.3909**
170	1.78E-005	2.9903	1.39E-005	2.9576	1.94E-006	2.8973	3.64E-06	4.3561
220	7.96E-006	3.1132	5.99E-006	3.2603	8.82E-007	3.0680	1.25E-06	4.1581
280	3.52E-006	3.3811	2.54E-006	3.5532	3.91E-007	3.3676	4.88E-07	3.8875
350	1.55E-006	3.6922	1.08E-006	3.8508	1.72E-007	3.6916	2.15E-07	3.6674
450	5.62E-007	4.0275	3.78E-007	4.1585	6.24E-008	4.0302	8.99E-08	3.4784
600	1.60E-007	4.3735	1.04E-007	4.4818	1.77E-008	4.3784	3.54E-08	3.2416
800	4.22E-008	**4.6264**	2.68E-008	4.7214	4.65E-009	4.6436	1.55E-08	2.8639
1000	1.49E-008	4.6621	9.13E-009	**4.8255**	1.62E-009	**4.7244**	9.16E-09	2.3633
1300	4.79E-009	4.3260	2.58E-009	4.8159	4.88E-010	4.5733	5.72E-09	1.7949
1600	2.26E-009	3.6228	9.53E-010	4.7989	1.92E-010	4.5041	4.37E-09	1.2969

Table.2: P=3 Primitive variables error vs. entropy error

# of grids	Density	(rate)	Velocity	(rate)	Pressure	(rate)	Entropy	(rate)
30	3.83E-004		2.14E-004		3.58E-005		2.80E-004	
60	3.74E-005	3.3524	3.06E-005	2.8081	4.65E-006	2.9468	2.35E-005	3.5779
100	8.03E-006	3.0140	6.09E-006	3.1622	9.56E-007	3.0939	2.93E-006	4.0702
130	3.36E-006	3.3184	2.36E-006	3.6081	3.93E-007	3.3894	1.04E-006	3.9582
170	1.23E-006	3.7407	8.26E-007	3.9152	1.43E-007	3.7724	3.11E-007	**4.4942**
220	4.34E-007	4.0510	2.89E-007	4.0807	4.99E-008	4.0788	1.14E-007	3.8876
280	1.41E-007	4.6549	9.71E-008	4.5143	1.61E-008	4.6914	6.39E-008	2.4063
350	4.27E-008	**5.3575**	2.76E-008	5.6358	4.57E-009	5.6449	5.30E-008	0.8437
450	1.67E-008	3.7330	5.61E-009	**6.3408**	9.49E-010	**6.2551**	4.56E-008	0.5943
600	1.27E-008	0.9665	9.14E-010	6.3080	2.21E-010	5.0688	3.76E-008	0.6725
800	1.03E-008	0.7322	4.50E-010	2.4638	1.67E-010	0.9712	3.05E-008	0.7312

It is apparent that errors in primitive variables yield a better measurement of the order of accuracy than Entropy errors. The reason for this effect is not yet understood. However, it is not straightforward to determine the exact solution in multiple dimensions. Hence, we still use entropy errors in 2-D and 3-D with the final time chosen to be 2 and gamma to be 3 to get optimal order of accuracy.

2-D

Fig.3: L2 (Entropy errors) vs. 1/sqrt(nDofs)

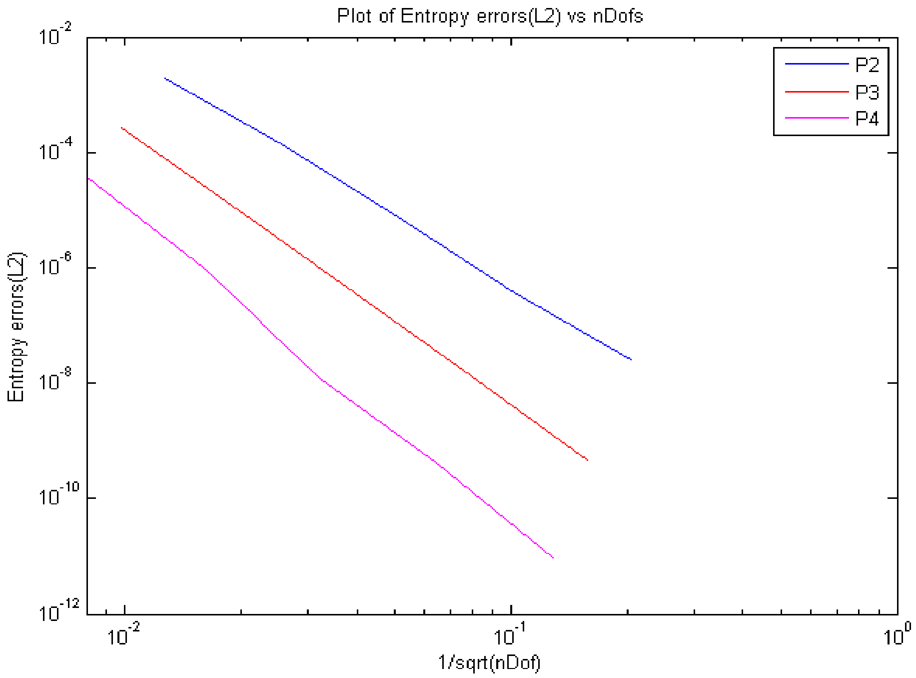
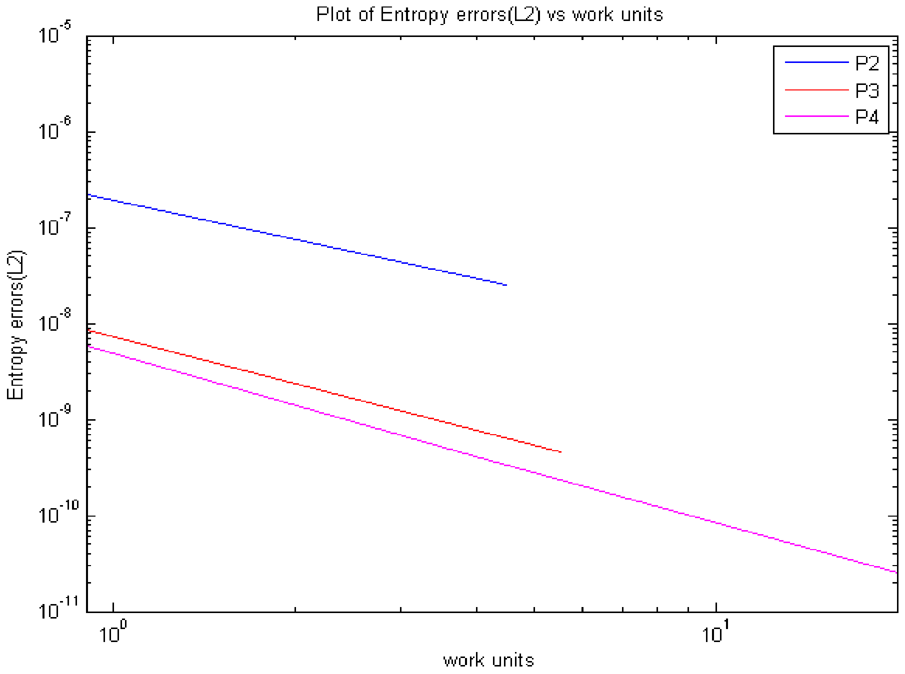


Fig.4: L2 (Entropy errors) vs. work units



Analysis of the order of accuracy

The order of accuracy for different orders of sub-cell representation is displayed in the table below. The time step was chosen carefully so that the spatial errors dominate over temporal errors.

Table.3:-

	Grid3	Grid4	Grid5	Grid6	Grid7
P2	-	3.7337	4.2335	4.3297	3.8703
P3	-	4.6453	4.8146	4.7675	4.8626
P4	-	5.2698	6.36614	4.8554	5.4224

It can be seen that for P2 the 2D code shows slightly better accuracy than 2P, but as the number of basis functions is increased, the order fails to reach the expected 2P + 1. This is attributed to the lack of sufficient smoothness in the problem at late times in multiple dimensions.

3D

Fig.5: L2 (Entropy errors) vs. 1/(nDofs)^(1/3)

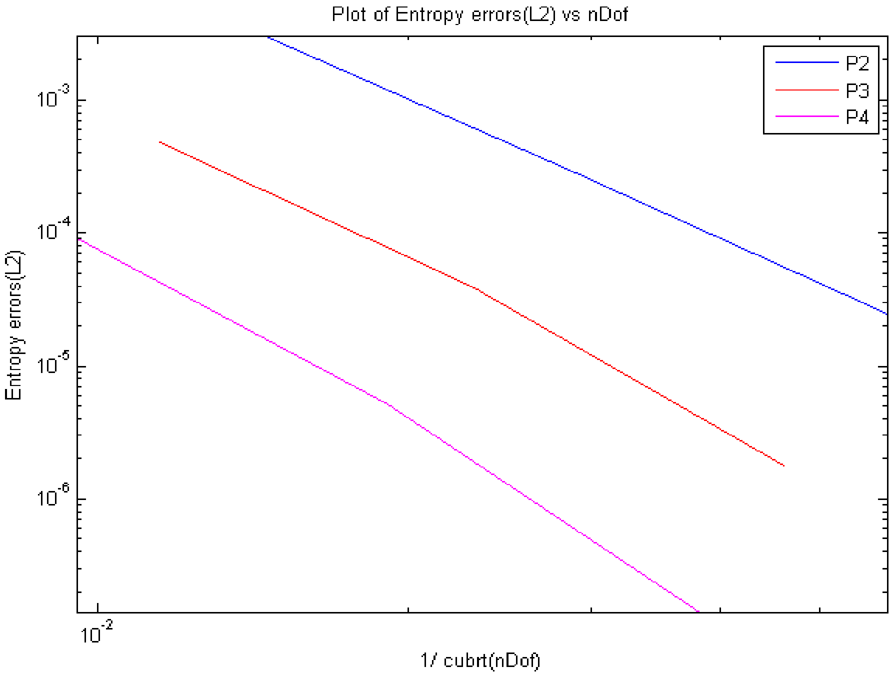
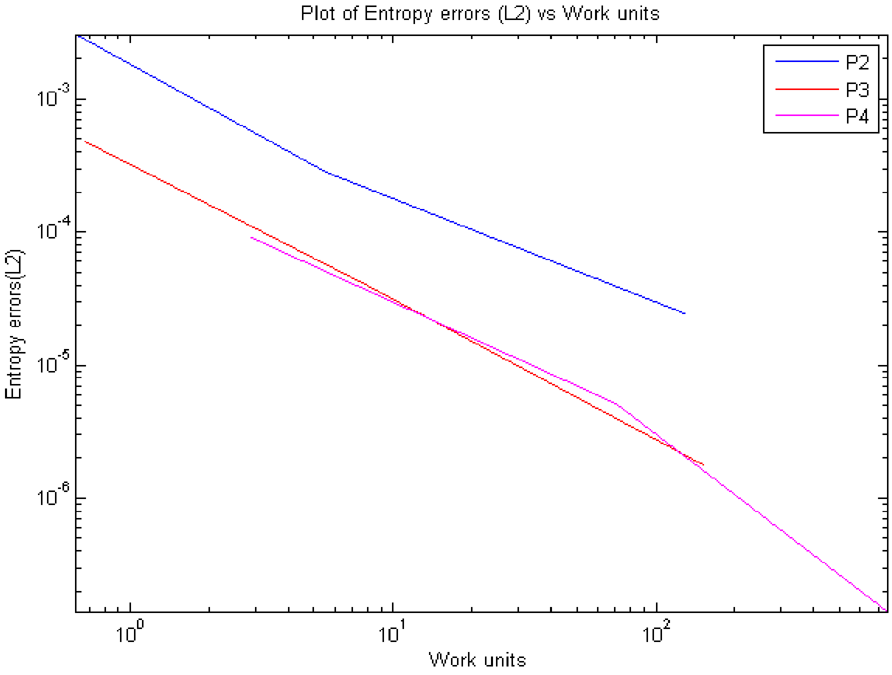


Fig.6: <u>L2 (Entropy errors) vs. Work units</u>



Analysis of the order of accuracy

Table.4:-

	Grid3	Grid4	Grid5
P2	-	3.4323	3.5295
P3	-	3.6080	4.4591
P4	-	4.1425	5.1987

The 3D results stay even farther below the theoretical order of accuracy than the 2D results. Non-smoothness of the underlying exact solution is again conjectured, but this issue needs further investigation. It would be useful to modify the expansion problem in 2D and 3D, perhaps by adding a smooth source term, so that the solution remains infinitely differentiable.