1st International Workshop on High-Order CFD Methods, Nashville, January 7-8, 2012

Summary of the C2.3 test case results

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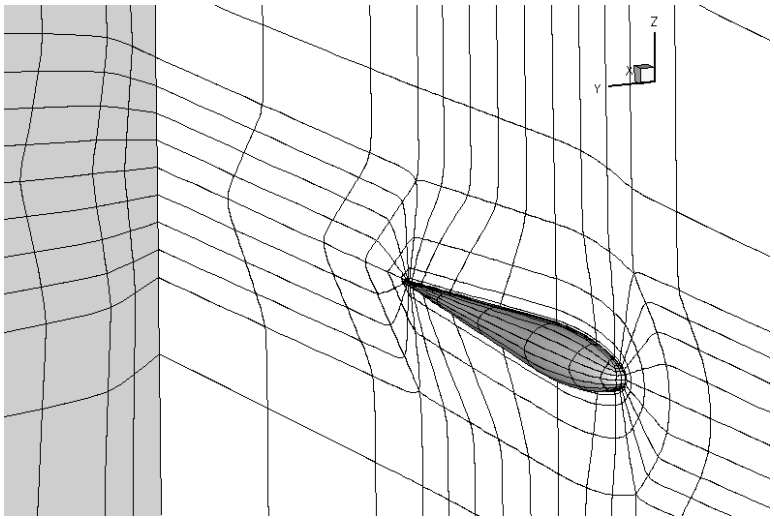
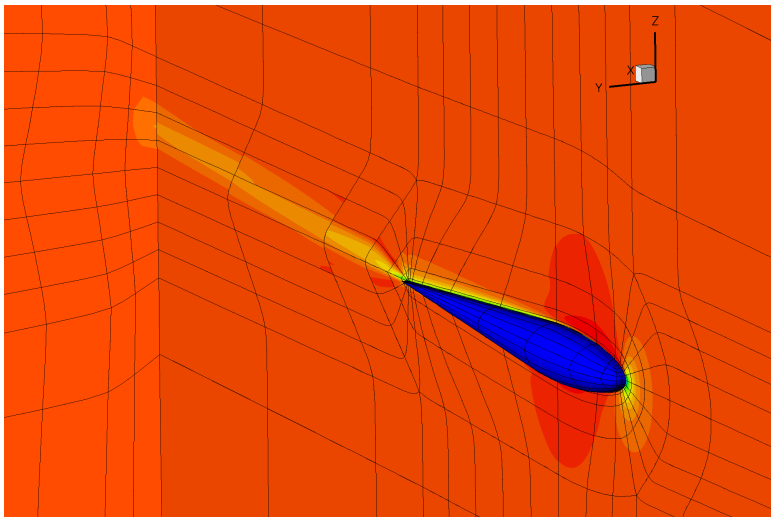
Test case C2.3

Compressible flow around a 3D body of revolution
(the BTC0 test case in the EU-project ADIGMA)

- inviscid: $M = 0.5$ and $\alpha = 1^\circ$
- laminar: $M = 0.5$, $Re = 5000$ and $\alpha = 1^\circ$
- turbulent: $M = 0.5$, $Re = 10 \cdot 10^6$ and $\alpha = 5^\circ$



btc0-NLR-T2.v2.m4.msh
with 6656 elements



DG($p = 3$) solution
for the RANS-$k\omega$ equations

Test case C2.3a

Inviscid flow at $M = 0.5$ and $\alpha = 1^\circ$ around the BTC0 geometry

Following p4 hexahredral grids have been provided on the workshop (hiocfd) homepage

- btc0-NLR-E1.v2.m4.msh 768 cells
- btc0-NLR-E2.v2.m4.msh 6,144 cells
- btc0-NLR-E3.v2.m4.msh 49,152 cells
- btc0-NLR-E4.v2.m4.msh 393,216 cells

"Official" convergence criterion (taken from "Notes for all participants")

Reduction of the density residual to 10^{-10} relative to freestream conditions measured in a normalized L^2-norm, i.e.

$$R/R_\infty < 10^{-10} \quad \text{for} \quad R = L^2(Res_\varrho) = \sqrt{\frac{1}{N}\sum_i^N Res_{\varrho,i}^2}$$

Summary of test case C2.3a

Inviscid flow at $M = 0.5$ and $\alpha = 1^\circ$ around the BTC0 geometry

Data available from

- Krzysztof Fidkowski, University of Michigan, **UMich**
- Ralf Hartmann, DLR, Braunschweig, **DLR**

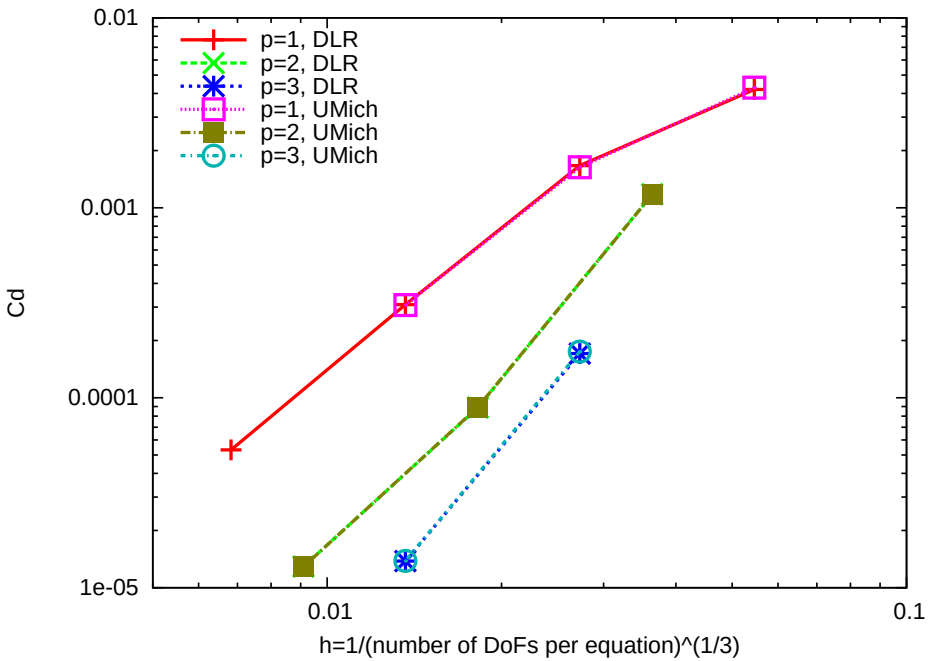
Case details

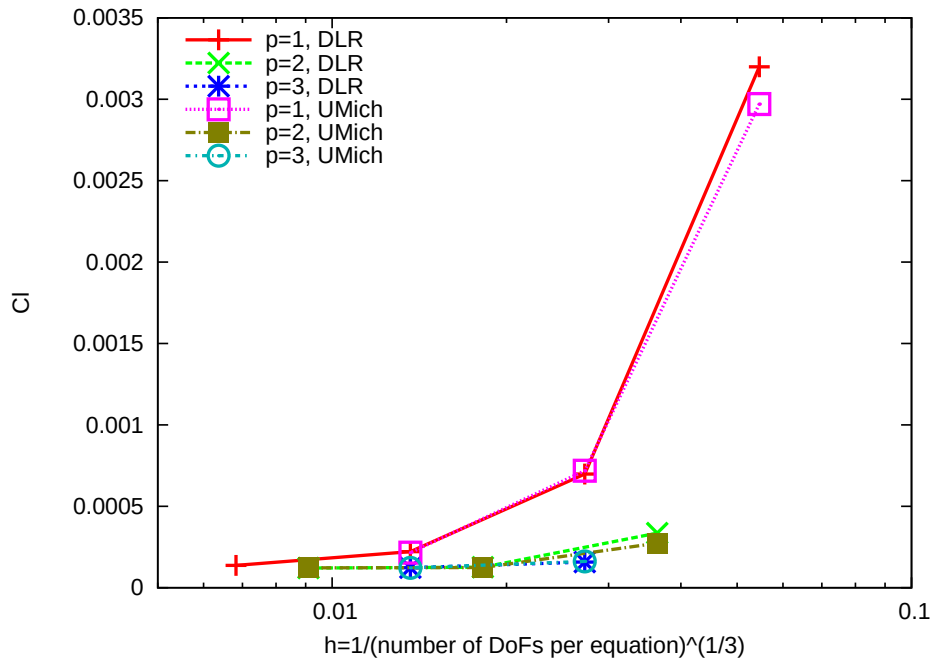
	UMich	**DLR**
discretization	DG	DG
numerical flux	Roe	Roe (with entropy fix)
farfield boundary	Roe and $\mathbf{u}_\infty$	characteristic
solver	Newton	Newton & backward Euler
grid	p4 grids (hiocfd)	p4 grids (hiocfd)
convergence	$R < 10^{-8}$	$R/R_\infty < 10^{-10}$
criterion	$\sum_i^N \lvert Res_{\varrho,i} \rvert$	$\sqrt{\frac{1}{N}\sum_i^N Res_{\varrho,i}^2}$

Note, that work units are not comparable due to different convergence criteria.

Summary of test case C2.3a

Inviscid flow at $M = 0.5$ and $\alpha = 1^\circ$ around the BTC0 geometry





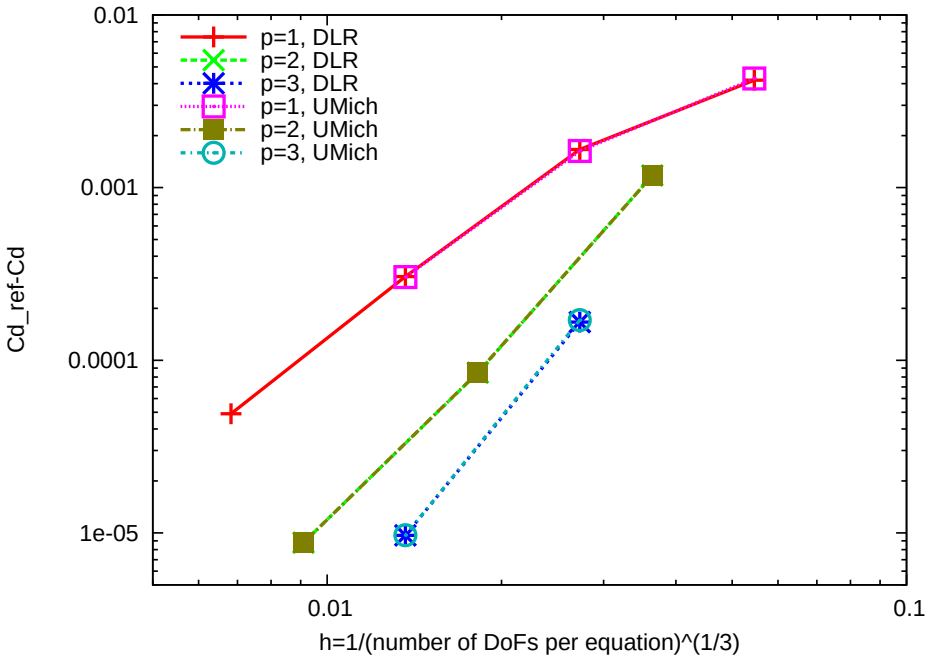
C_d (left) and C_l (right) vs. $h = 1/\sqrt[3]{\#\text{ DoFs per equation}}$ for $p = 1, \ldots, 3$

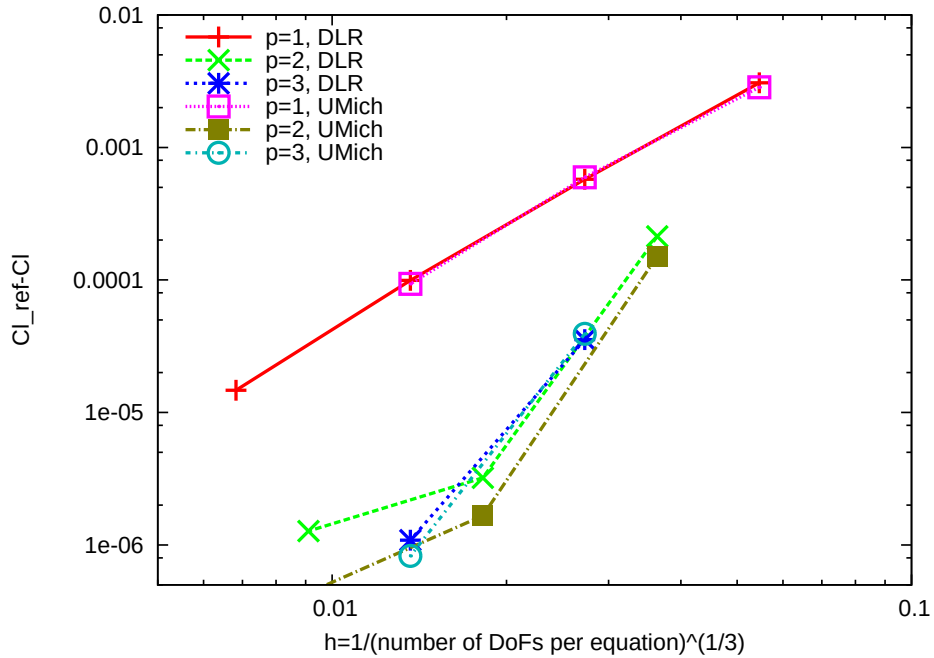
Summary of test case C2.3a

Inviscid flow at $M = 0.5$ and $\alpha = 1^\circ$ around the BTC0 geometry

Reference values (taken from the p=2 solution of DLR on 393 216 elements and $\sim 53 \cdot 10^6$ DoFs):

$$C_d^{\text{ref}} = 4.1402 \cdot 10^{-6},\ C_l^{\text{ref}} = 0.0001222976$$





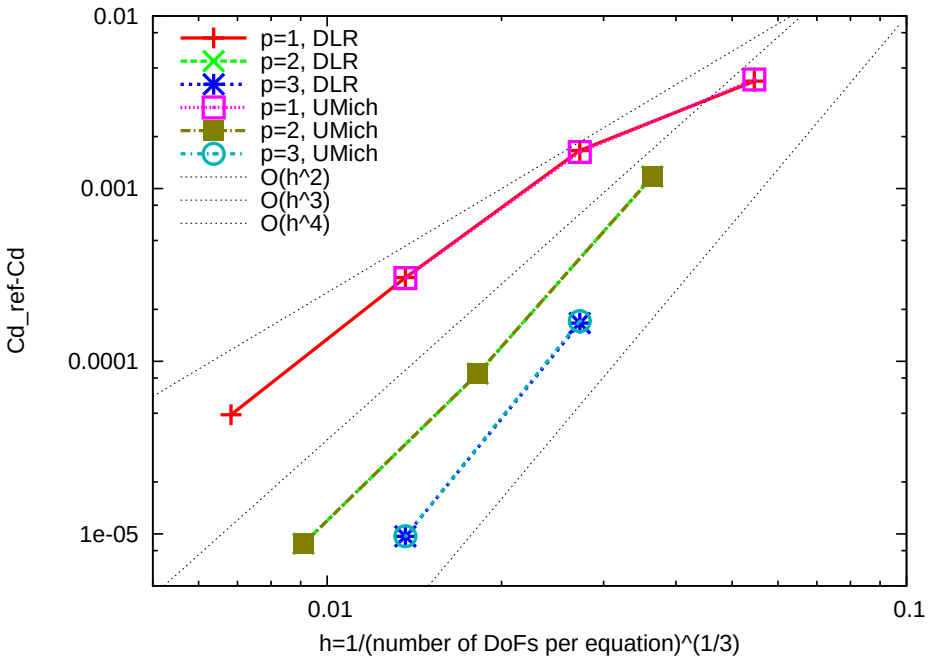
Error in C_d (left) and C_l (right) vs. $h = 1/\sqrt[3]{\#\text{ DoFs per equation}}$ for $p = 0, \ldots, 3$

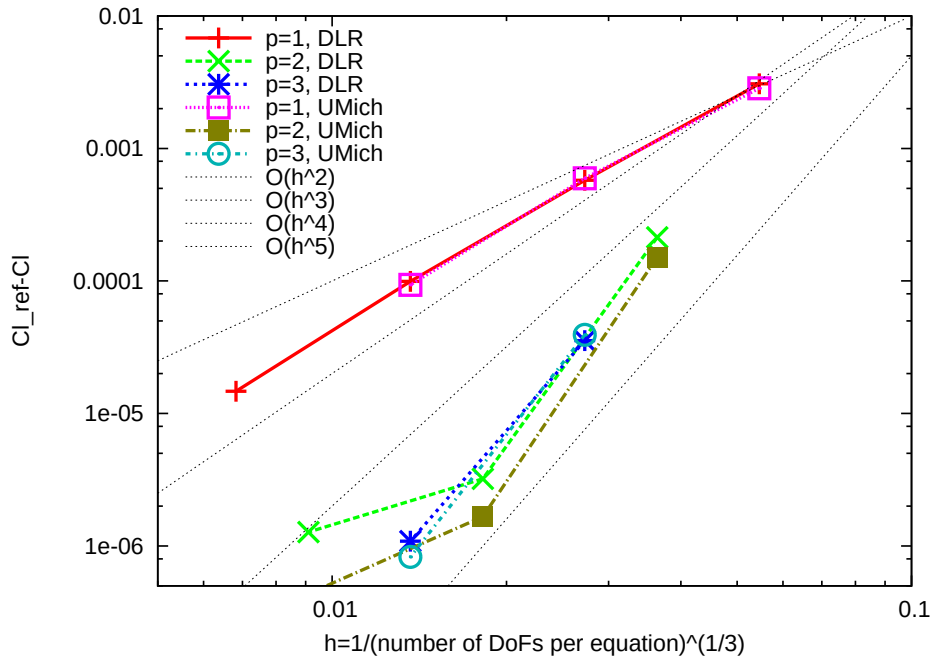
Summary of test case C2.3a

Inviscid flow at $M = 0.5$ and $\alpha = 1^\circ$ around the BTC0 geometry

Reference values (taken from the p=2 solution of DLR on 393 216 elements and $\sim 53 \cdot 10^6$ DoFs):

$$C_d^{\text{ref}} = 4.1402 \cdot 10^{-6},\ C_l^{\text{ref}} = 0.0001222976$$





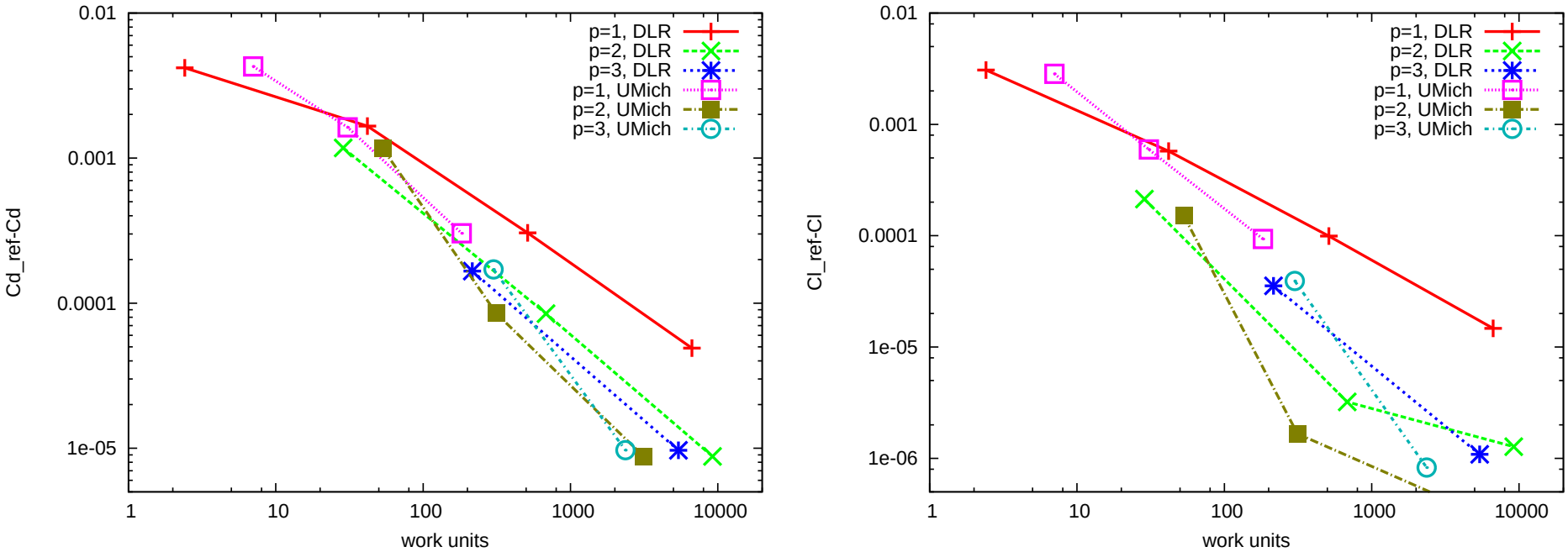
Error in C_d (left) and C_l (right) vs. $h = 1/\sqrt[3]{\#\text{ DoFs per equation}}$ for $p = 0, \ldots, 3$

Summary of test case C2.3a

Inviscid flow at $M = 0.5$ and $\alpha = 1°$ around the BTC0 geometry

Reference values (taken from the p=2 solution of DLR on 393 216 elements and $\sim 53 \cdot 10^6$ DoFs):

$$C_d^{\text{ref}} = 4.1402 \cdot 10^{-6},\ C_l^{\text{ref}} = 0.0001222976$$



Error in C_d (left) and C_l (right) vs. work units for $p = 0, \ldots, 3$

Note, that work units are not comparable due to different convergence criteria.

Test case C2.3b

Laminar flow at $M = 0.5$, $Re = 5000$ and $\alpha = 1^\circ$ around the BTC0 geometry

Following p4 hexahredral grids have been provided on the workshop (hiocfd) homepage

- btc0-NLR-L1.v2.m4.msh 768 cells
- btc0-NLR-L2.v2.m4.msh 6,144 cells
- btc0-NLR-L3.v2.m4.msh 49,152 cells
- btc0-NLR-L4.v2.m4.msh 393,216 cells

"Official" convergence criterion (taken from "Notes for all participants")

Reduction of the density residual to 10^{-10} relative to freestream conditions measured in following L^2-norm, i.e.

$$R/R_\infty < 10^{-10} \quad \text{for} \quad R = L^2(Res_\varrho) = \sqrt{\frac{1}{N}\sum_i^N Res_{\varrho,i}^2}$$

Summary of test case C2.3b

Laminar flow at $M = 0.5$, $Re = 5000$ and $\alpha = 1^\circ$ around the BTC0 geometry

Data available from

- Krzysztof Fidkowski, University of Michigan, **UMich**
- Ralf Hartmann, DLR, Braunschweig, **DLR**

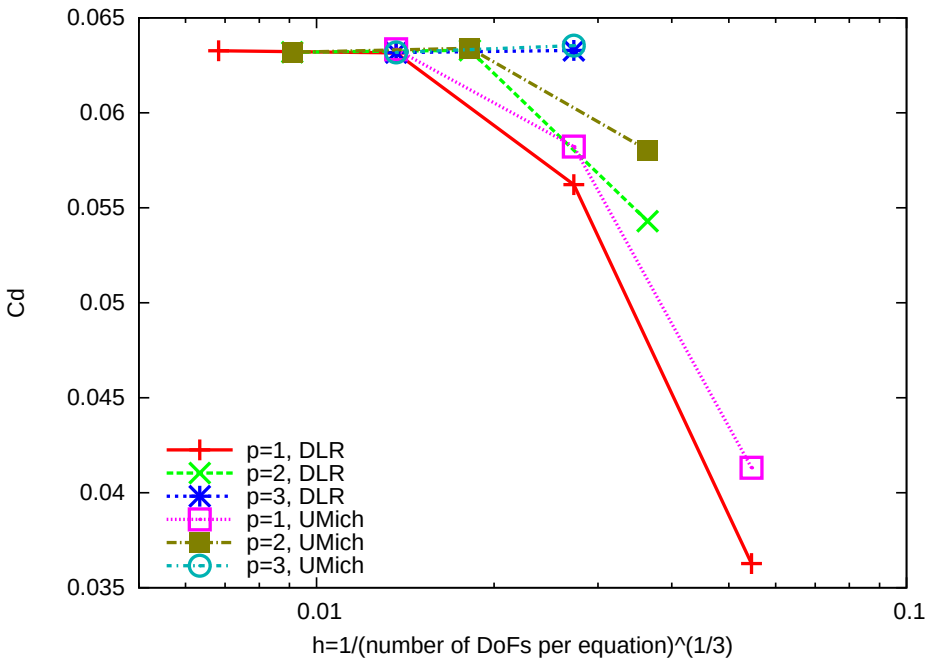
Case details

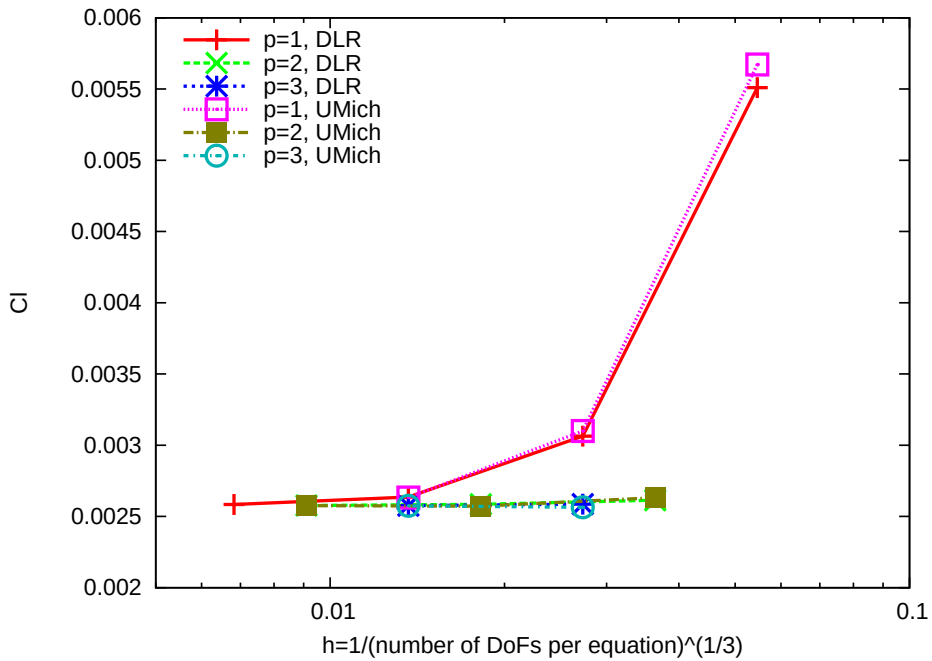
	UMich	**DLR**
discretization	DG, BR2(?)	DG, BR2
numerical flux	Roe	Roe (with entropy fix)
farfield boundary	Roe and $\mathbf{u}_\infty$	characteristic
solver	Newton	Newton
grid	p4 grids (hiocfd)	p4 grids (hiocfd)
convergence	$R < 10^{-8}$	$R/R_\infty < 10^{-10}$
criterion	$\sum_i^N \lvert Res_{\varrho,i} \rvert$	$\sqrt{\frac{1}{N}\sum_i^N Res_{\varrho,i}^2}$

Note, that work units are not comparable due to different convergence criteria.

Summary of test case C2.3b

Laminar flow at $M = 0.5$, $Re = 5000$ and $\alpha = 1^\circ$ around the BTC0 geometry





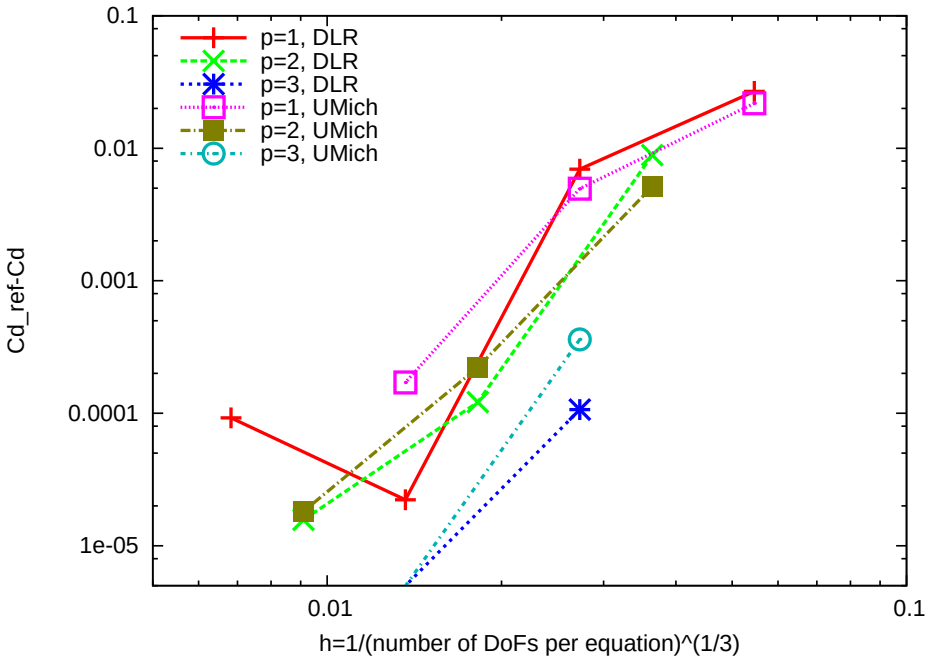
C_d (left) and C_l (right) vs. $h = 1/\sqrt[3]{\#\text{ DoFs per equation}}$ for $p = 0, \ldots, 3$

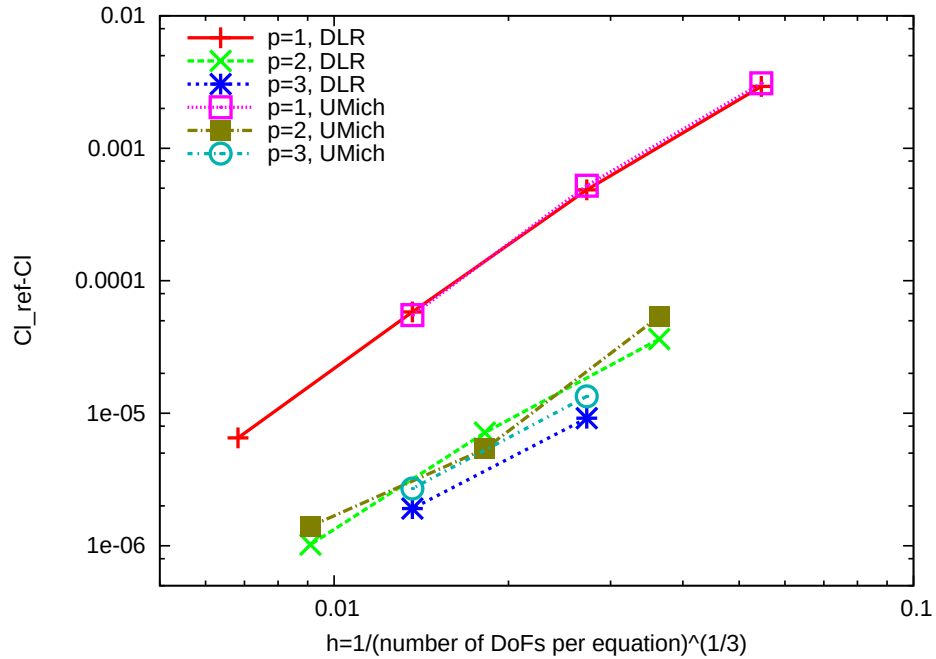
Summary of test case C2.3b

Laminar flow at $M = 0.5$, $Re = 5000$ and $\alpha = 1°$ around the BTC0 geometry

Reference values (taken from the p=2 solution of DLR on 393 216 elements and $\sim 53 \cdot 10^6$ DoFs):

$$C_d^{\text{ref}} = 0.0631734067,\ C_l^{\text{ref}} = 0.0025778642875$$





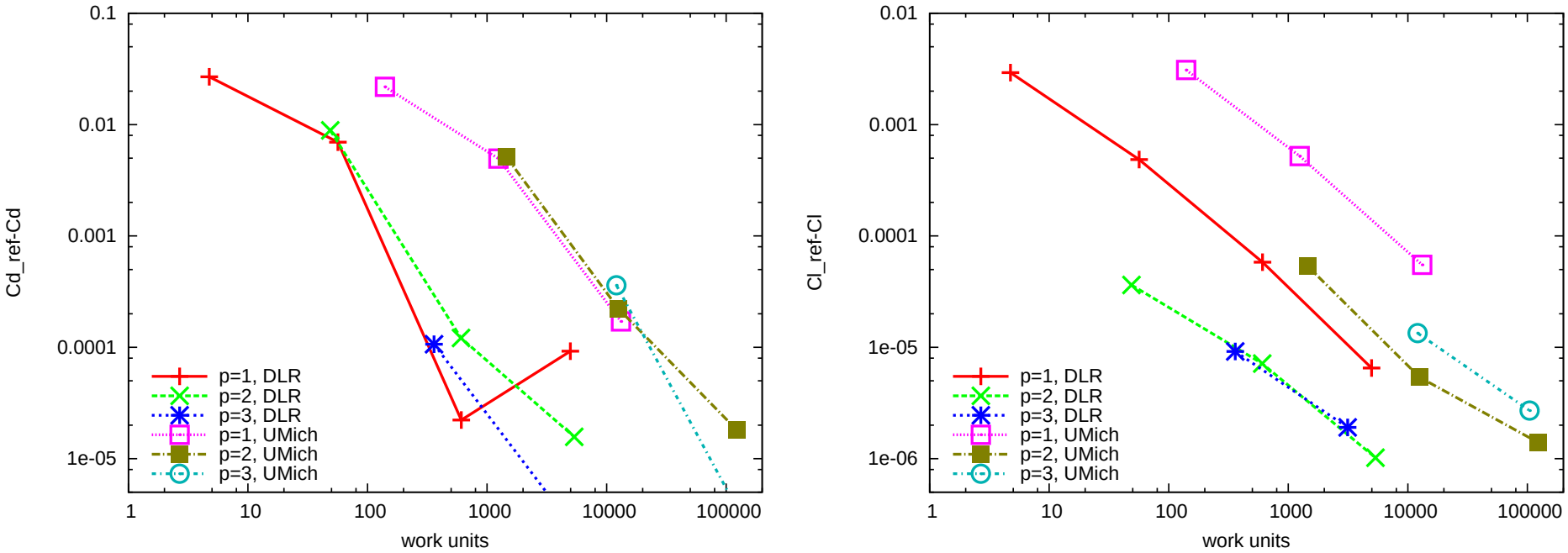
Error in C_d (left) and C_l (right) vs. $h = 1/\sqrt[3]{\#\text{ DoFs per equation}}$ for $p = 0, \ldots, 3$

Summary of test case C2.3b

Laminar flow at $M = 0.5$, $Re = 5000$ and $\alpha = 1^\circ$ around the BTC0 geometry

Reference values (taken from the p=2 solution of DLR on 393 216 elements and $\sim 53 \cdot 10^6$ DoFs):

$$C_d^{\text{ref}} = 0.0631734067,\ C_l^{\text{ref}} = 0.0025778642875$$



Error in C_d (left) and C_l (right) vs. work units for $p = 0, \ldots, 3$

Test case C2.3c

Turbulent flow at $M = 0.5$, $Re = 10 \cdot 10^6$ and $\alpha = 5°$ around the BTC0 geometry

Following p4 hexahredral grids have been provided on the workshop (hiocfd) homepage

- btc0-NLR-T1.v2.m4.msh 768 cells
- btc0-NLR-T2.v2.m4.msh 6,656 cells
- btc0-NLR-T3.v2.m4.msh 53,248 cells
- btc0-NLR-T4.v2.m4.msh 425,984 cells

"Official" convergence criterion (taken from "Notes for all participants")

Reduction of the density residual to 10^{-10} relative to freestream conditions measured in a normalized L^2-norm, i.e.

$$R/R_\infty < 10^{-10} \quad \text{for} \quad R = L^2(Res_\varrho) = \sqrt{\frac{1}{N}\sum_i^N Res_{\varrho,i}^2}$$

Summary of test case C2.3c

Turbulent flow at $M = 0.5$, $Re = 10 \cdot 10^6$ and $\alpha = 5°$ around the BTC0 geometry

Data available from

- Ralf Hartmann, DLR, Braunschweig, **DLR**

Case details

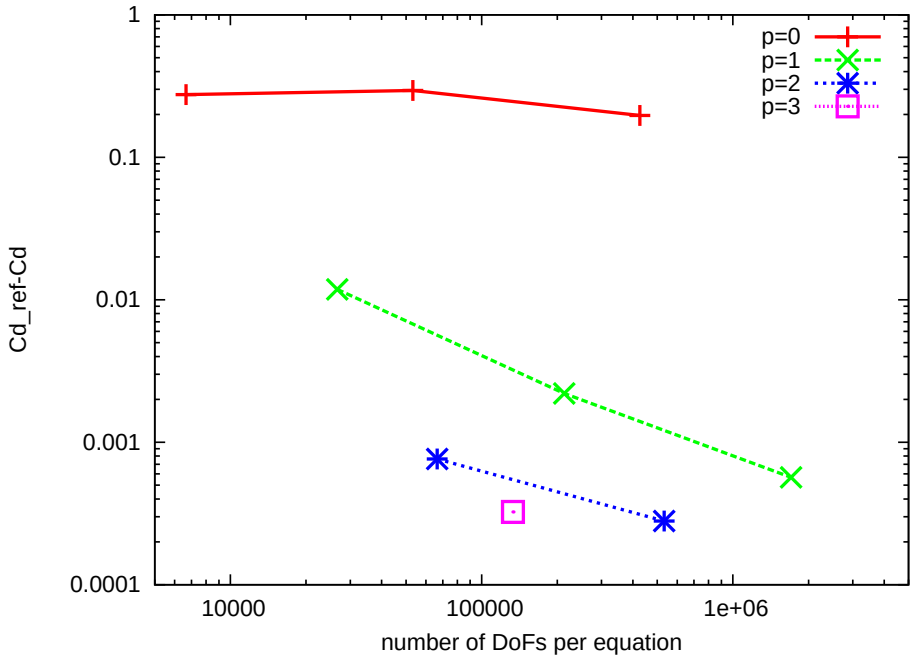
	DLR
equations	RANS-$k\omega$
discretization	DG, BR2
numerical flux	local Lax Friedrichs
farfield boundary	characteristic
solver	backward Euler
grid	p4 grids (hiocfd)
convergence	$R/R_\infty < 10^{-10}$
criterion	$\sqrt{\frac{1}{N}\sum_i^N Res_{\varrho,i}^2}$

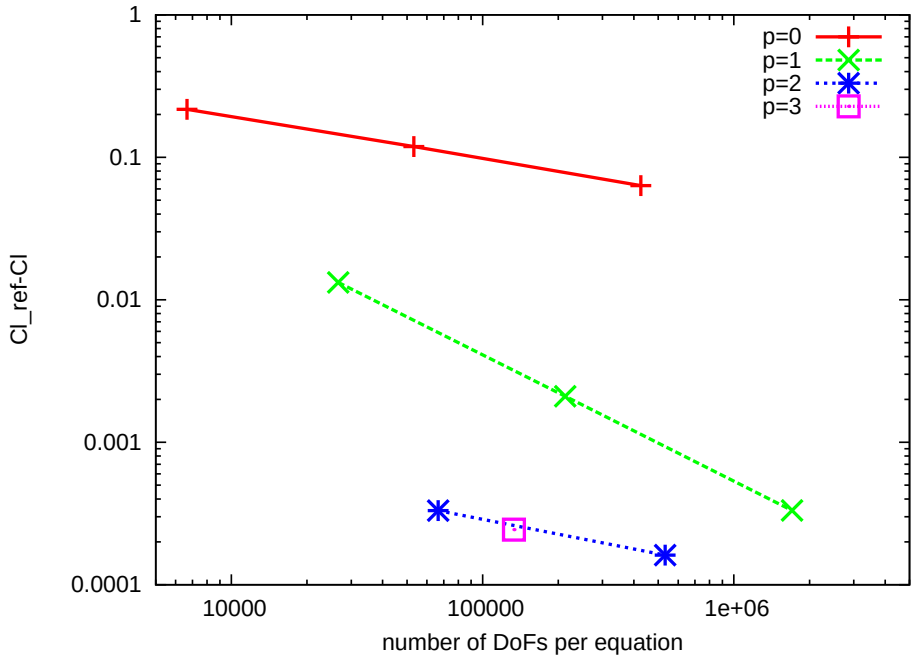
Summary of test case C2.3c

Turbulent flow at $M = 0.5$, $Re = 10 \cdot 10^6$ and $\alpha = 5^\circ$ around the BTC0 geometry

Reference values (taken from [HHL11]):

$$C_d^{\text{ref}} = 0.00835, \text{ and } C_l^{\text{ref}} = 0.006612$$





Error in C_d (left) and C_l (right) vs. number of DoFs per equation for $p = 0, \ldots, 3$

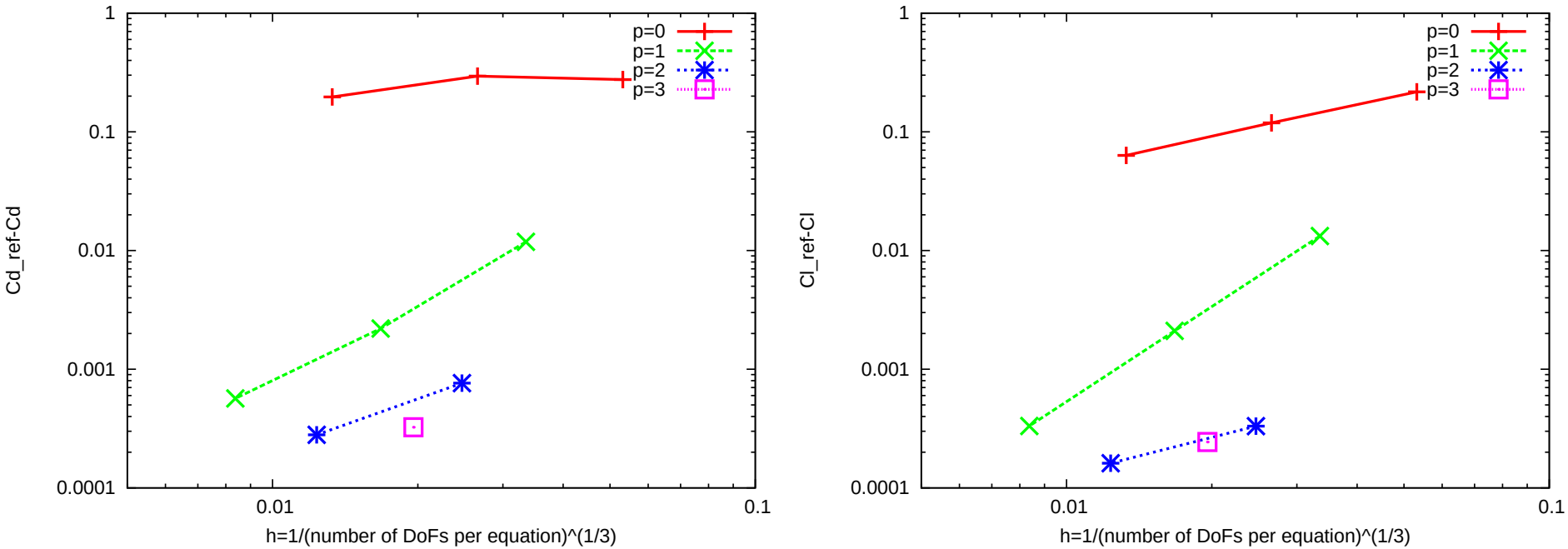
[HHL11] R. Hartmann, J. Held and T. Leicht Adjoint-based error estimation and adaptive mesh refinement for the RANS and k- turbulence model equations J. Comput. Phys., 230(11): 4268-4284, 2011.

Summary of test case C2.3c

Turbulent flow at $M = 0.5$, $Re = 10 \cdot 10^6$ and $\alpha = 5^\circ$ around the BTC0 geometry

Reference values (taken from [HHL11]):

$$C_d^{\text{ref}} = 0.00835, \text{ and } C_l^{\text{ref}} = 0.006612$$



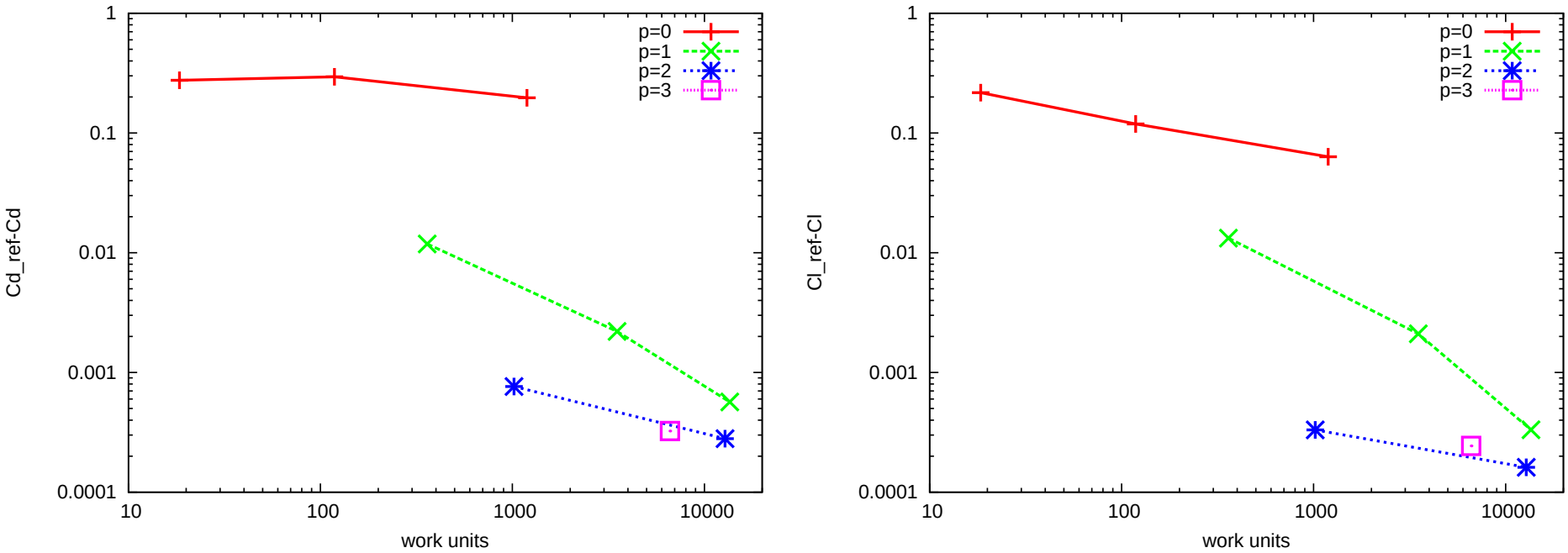
Error in C_d (left) and C_l (right) vs. $h = 1/\sqrt[3]{\#\text{ DoFs per equation}}$ for $p = 0, \ldots, 3$

Summary of test case C2.3c

Turbulent flow at $M = 0.5$, $Re = 10 \cdot 10^6$ and $\alpha = 5^\circ$ around the BTC0 geometry

Reference values (taken from [HHL11]):

$$C_d^{\text{ref}} = 0.00835, \text{ and } C_l^{\text{ref}} = 0.006612$$



Error in C_d (left) and C_l (right) vs. work units for $p = 0, \ldots, 3$

Overall summary of test case C2.3

	C2.3a(inviscid)	C2.3b(laminar)	C2.3c(turbulent)
UMich	results	results	
DLR	results	results	results

Overall comparison of accuracy:

- Test case C2.3a(inviscid): Extremely good agreement in C_d and C_l
- Test case C2.3b(laminar): Good agreement in C_l, C_d(UMich) on the same mesh more accurate than C_d(DLR)

Overall comparison of computing time/work units:

No exact comparison possible, due to different convergence criteria.
From the data available

- Test case C2.3a(inviscid): UMich more work units than DLR on coarse mesh but less on finer ones
- Test case C2.3b(laminar): UMich required a factor of more than 10 more work units than DLR (due to parallelization? due to convergence criterion?)