Cenaero

C3.5 –Taylor-Green vortex at Re=1600
Summary of results / Energy balance

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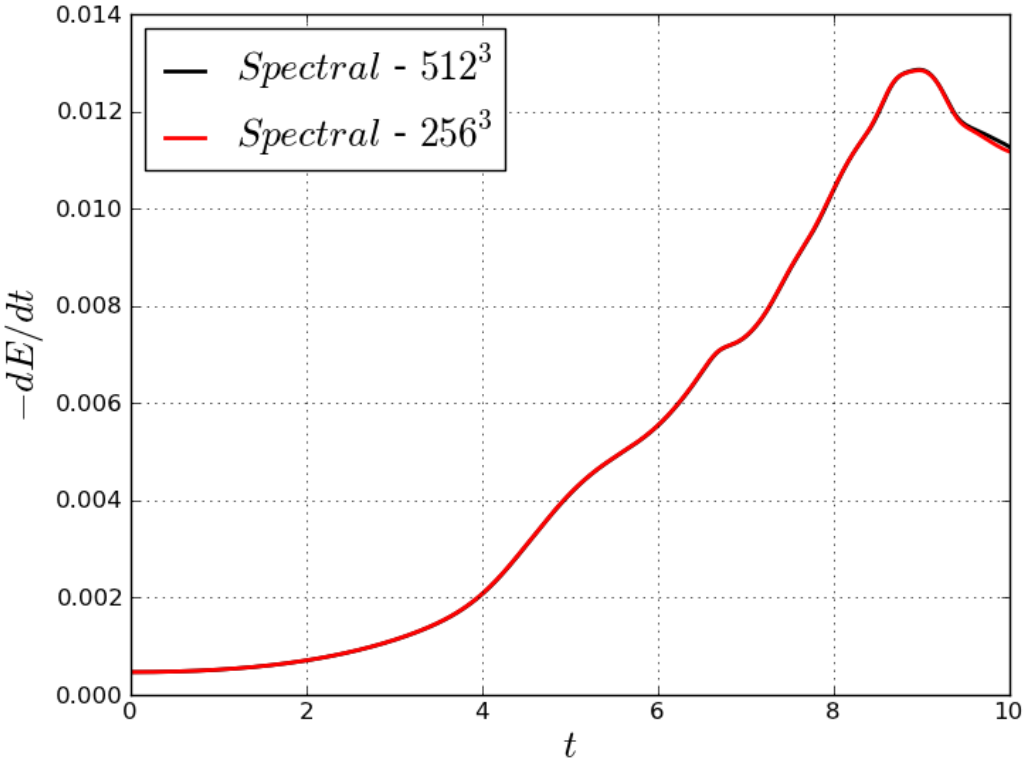
Flow physics

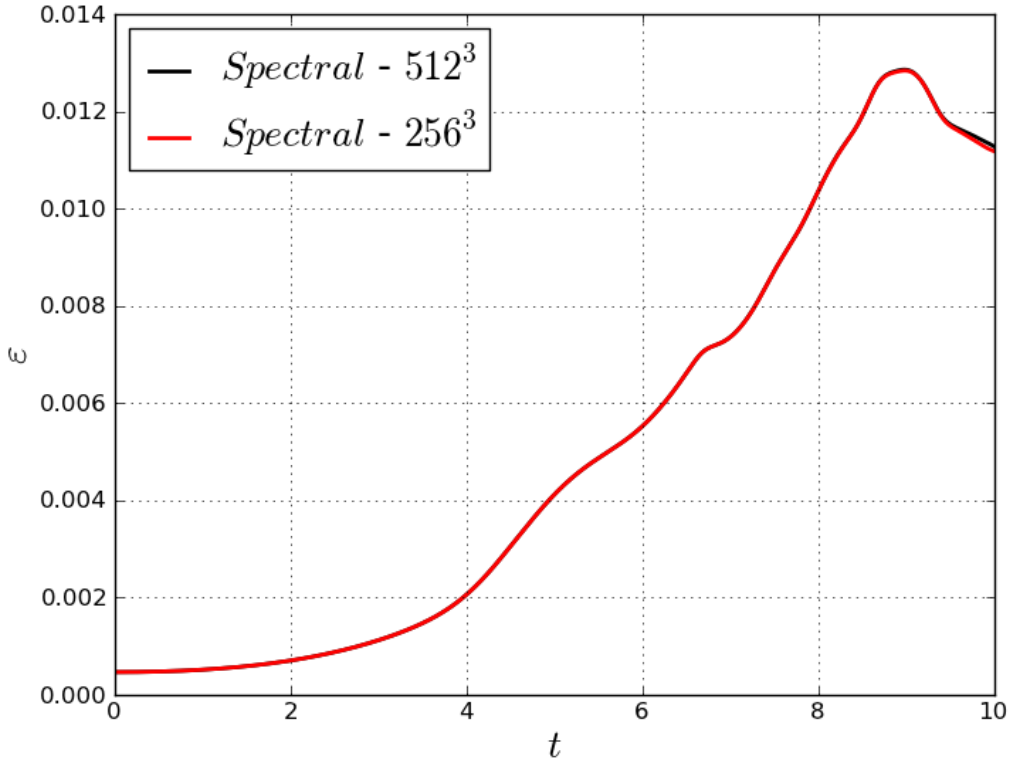
- (nearly) incompressible flow
- analytical initial solution
- transition to turbulence

- **Comparison to resolved computation with spectral code (512^3) / M. Duponcheel UCL**
  - **Evolution of the energy**
  - **Evolution of the dissipation (near peak)**
  - **Flow field at t=8 (near peak)**
- **Theoretical dissipation (incompressible) → measure for numerical dissipation**

$$\frac{dE}{dt} = -2\nu \int_V \frac{1}{2}\boldsymbol{\omega} \cdot \boldsymbol{\omega} \; dV$$

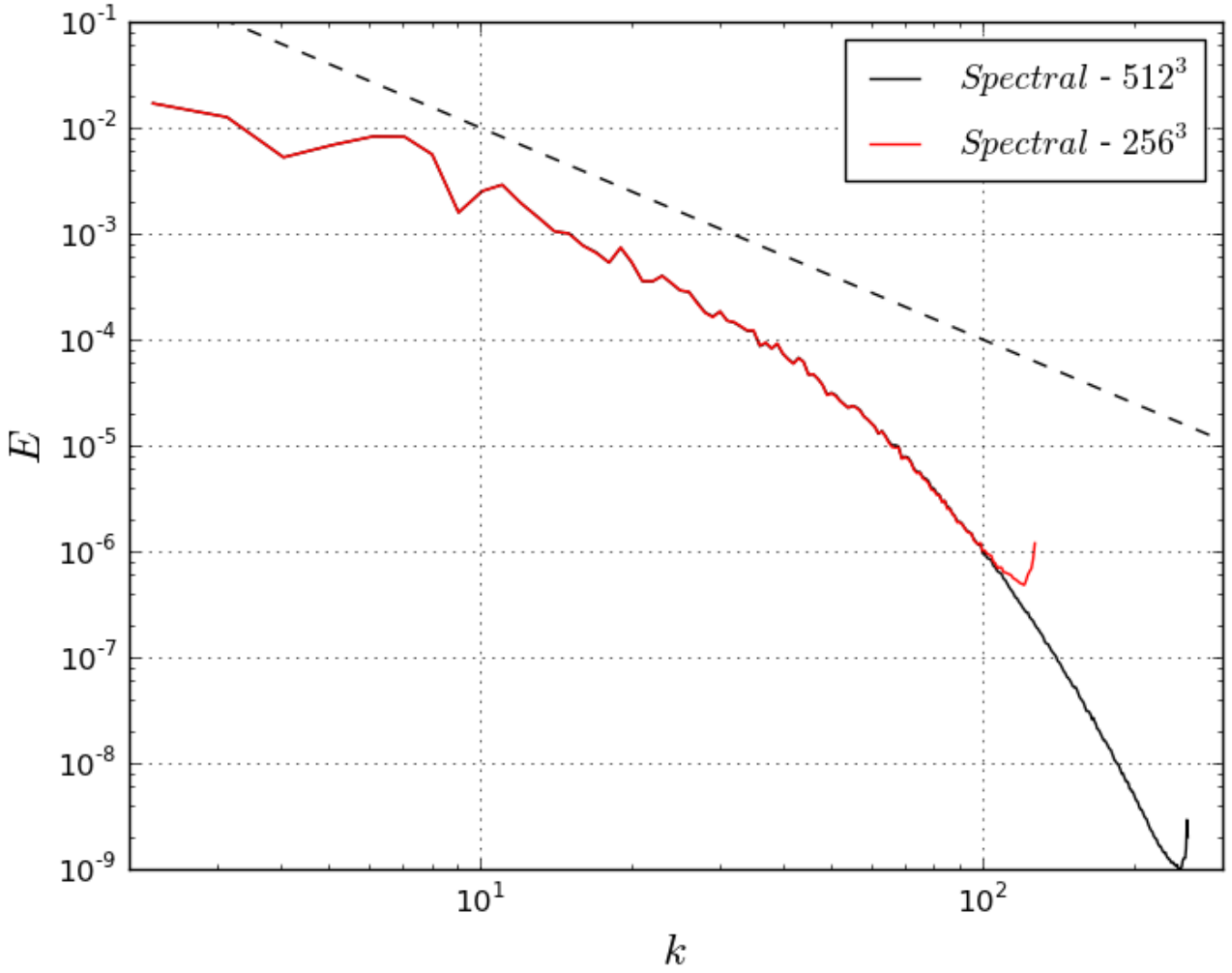
Reference results – enstrophy and dissipation





Flow structure at t=8

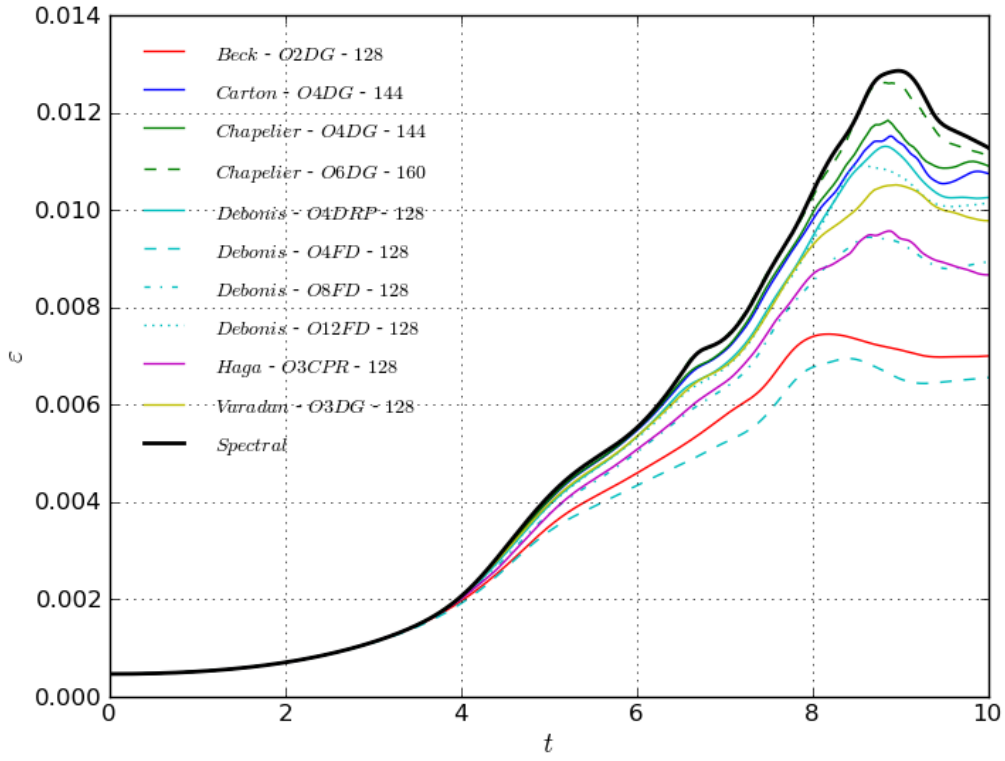
Reference results – energy spectrum at t=9

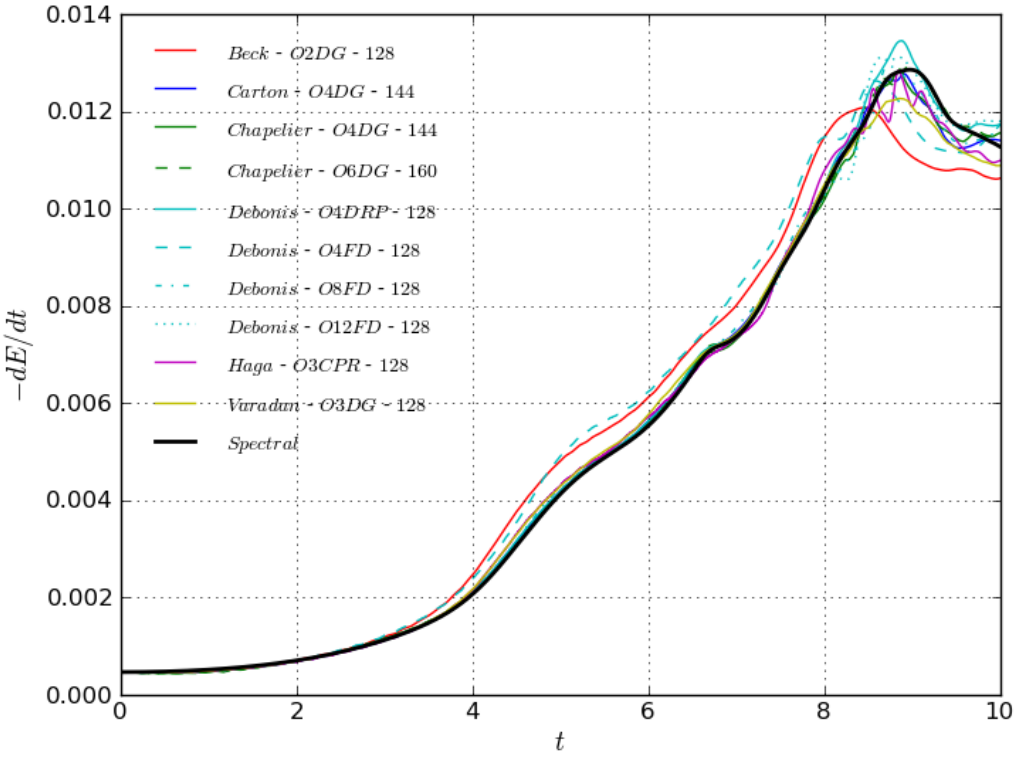


Overview

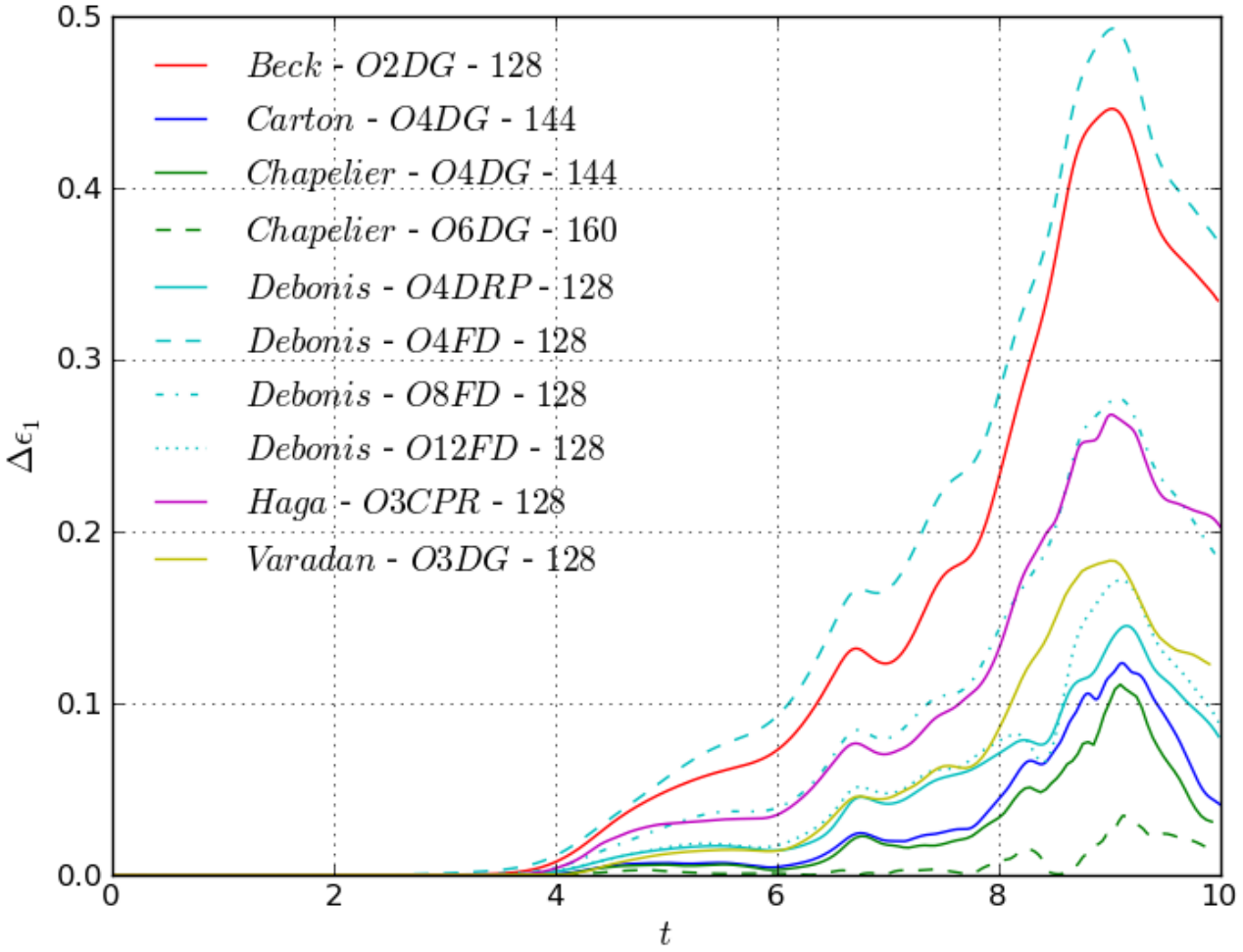
Participant	Institution	Method	Time	Resolution
				*Order / resolution / dt * 1000*
A. Beck G. Gassner	IAG/UniStuttgart	DGSEM	RK5	2/128/4,4/192/3.2,8/224/3.1, 10/225/3,16/240/2
C. Carton K.Hillewaert	Cenaero	DG/IP	RK4	4/144/4, 4/192/, 4/288, 4/384, (5/192), (6/190)
J.B. Chapelier	Onera	DG/BR2	RK4?	4/192/7.3, 4/288/4.85
J. Debonis	NASA Glenn	FD	RK4	(4,8,12) x (64,128,256) x 1.7
		FD / DRP	RK4	(4) x (64,128,256) x 1.7
T. Haga	Iowa State	DG-CRP	RK4?	3/128/4, 4/192/4, 5/256/2, 3/192/4
J.M. Le Gouez	Onera	FV/Recon	RK3?	3/220, 4/220, 4/292
S. Varadan	Uni Michigan	DG/RDG	RK4	3/128

Theoretical and measured dissipation / 128

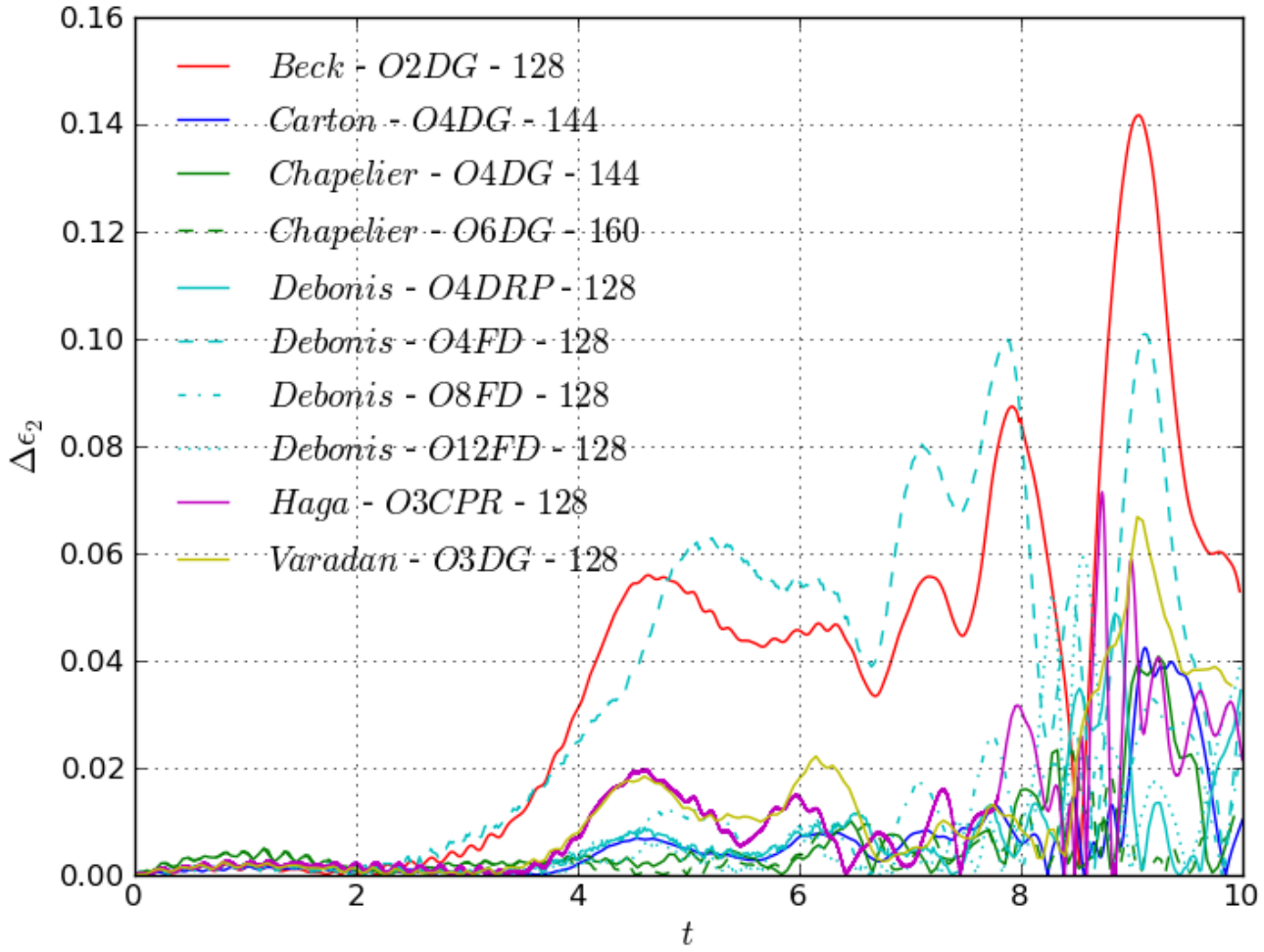




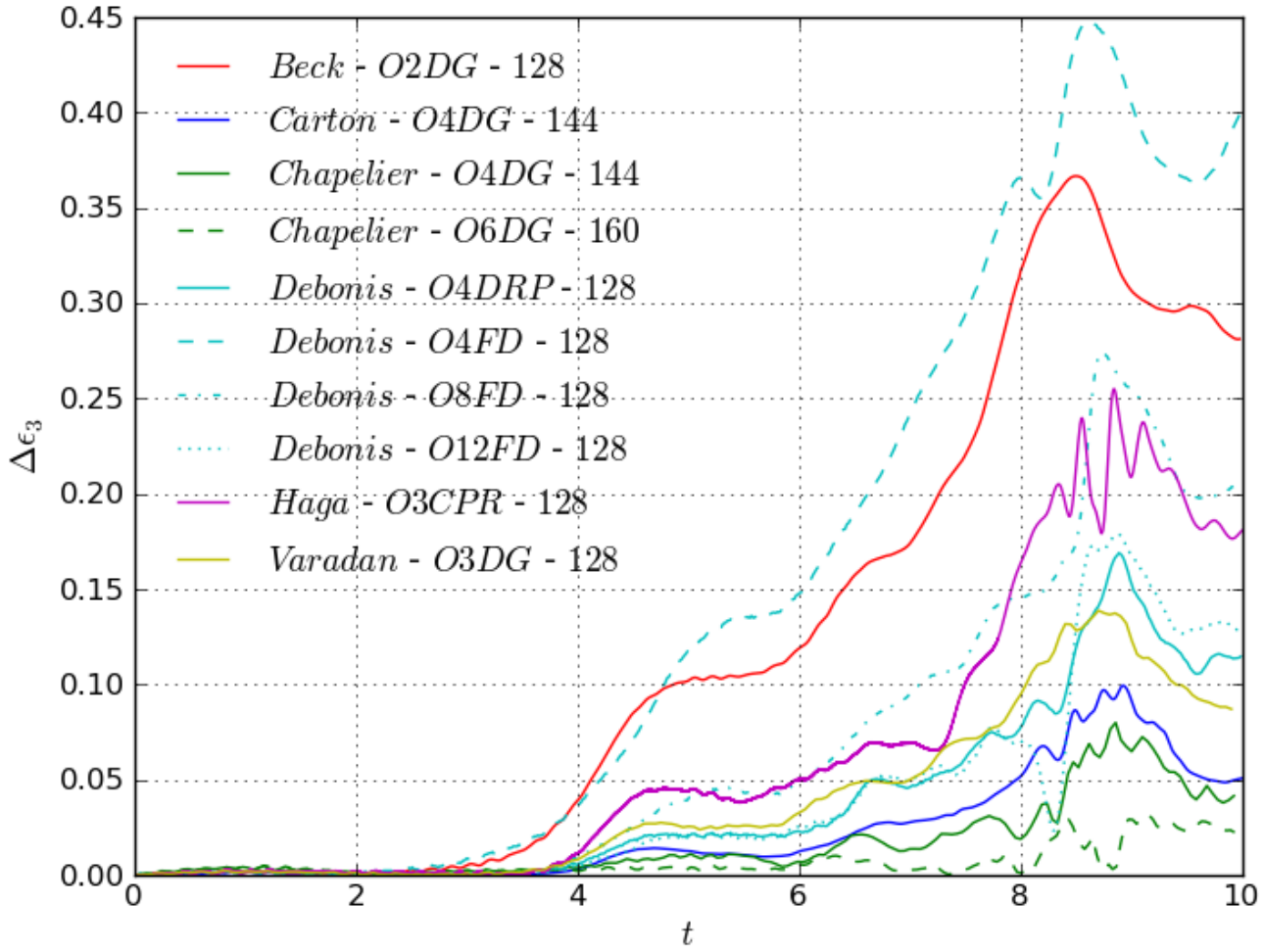
Error on theoretical dissipation / 128



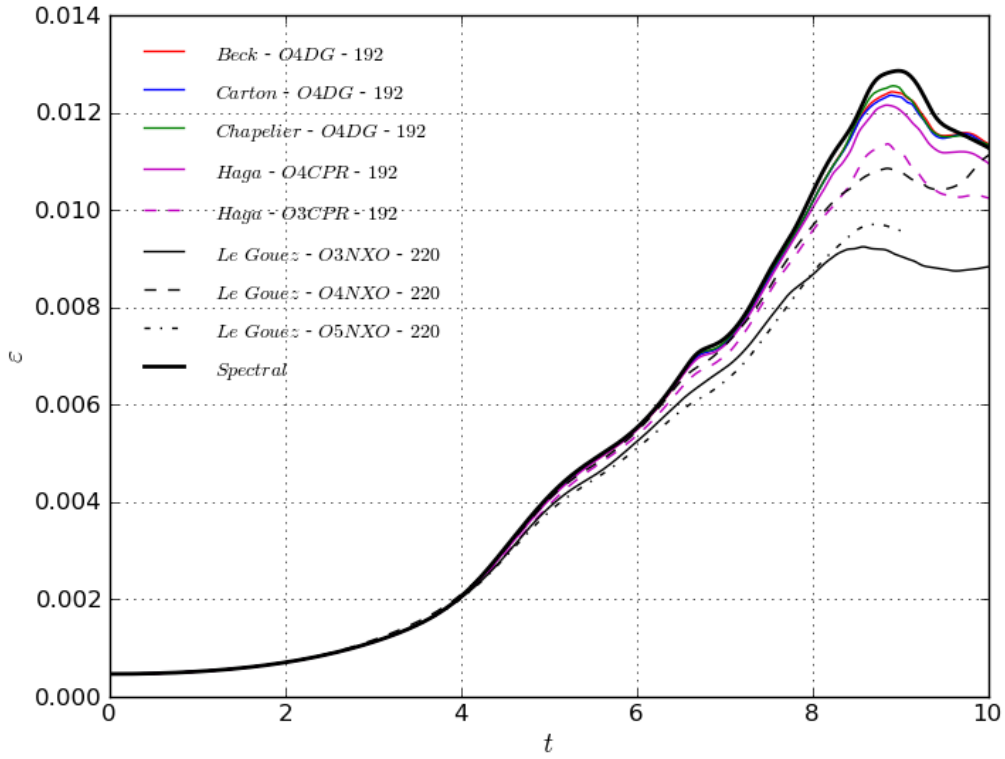
Error on measured dissipation / 128

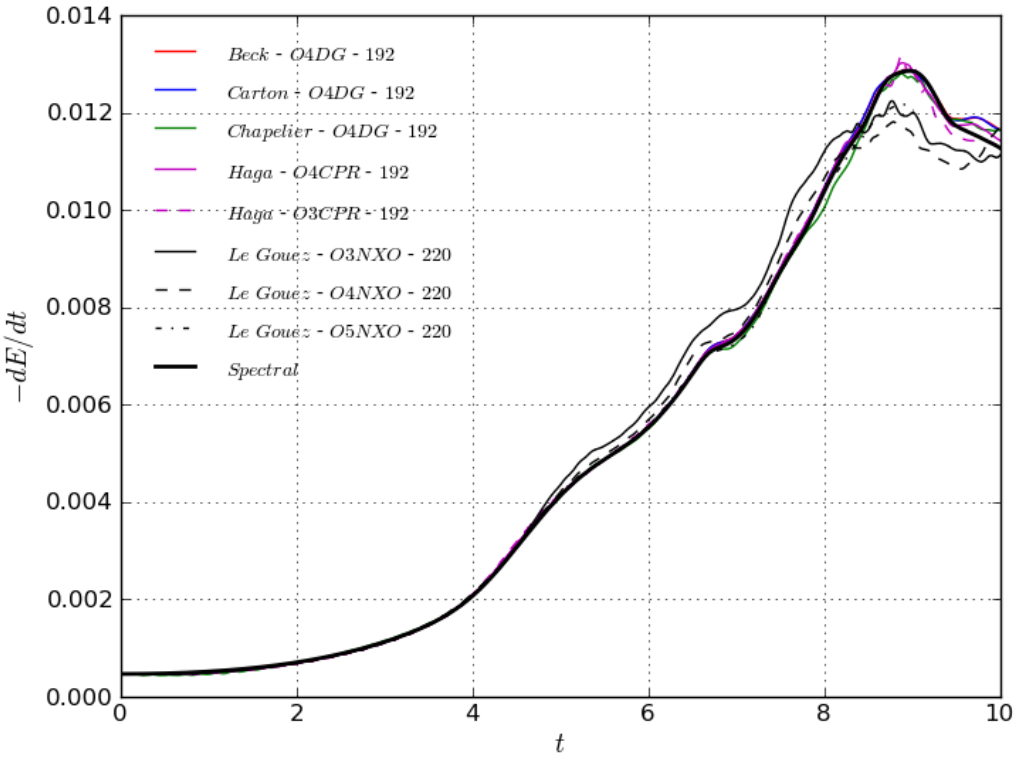


Difference theoretical & measured dissipation / 128

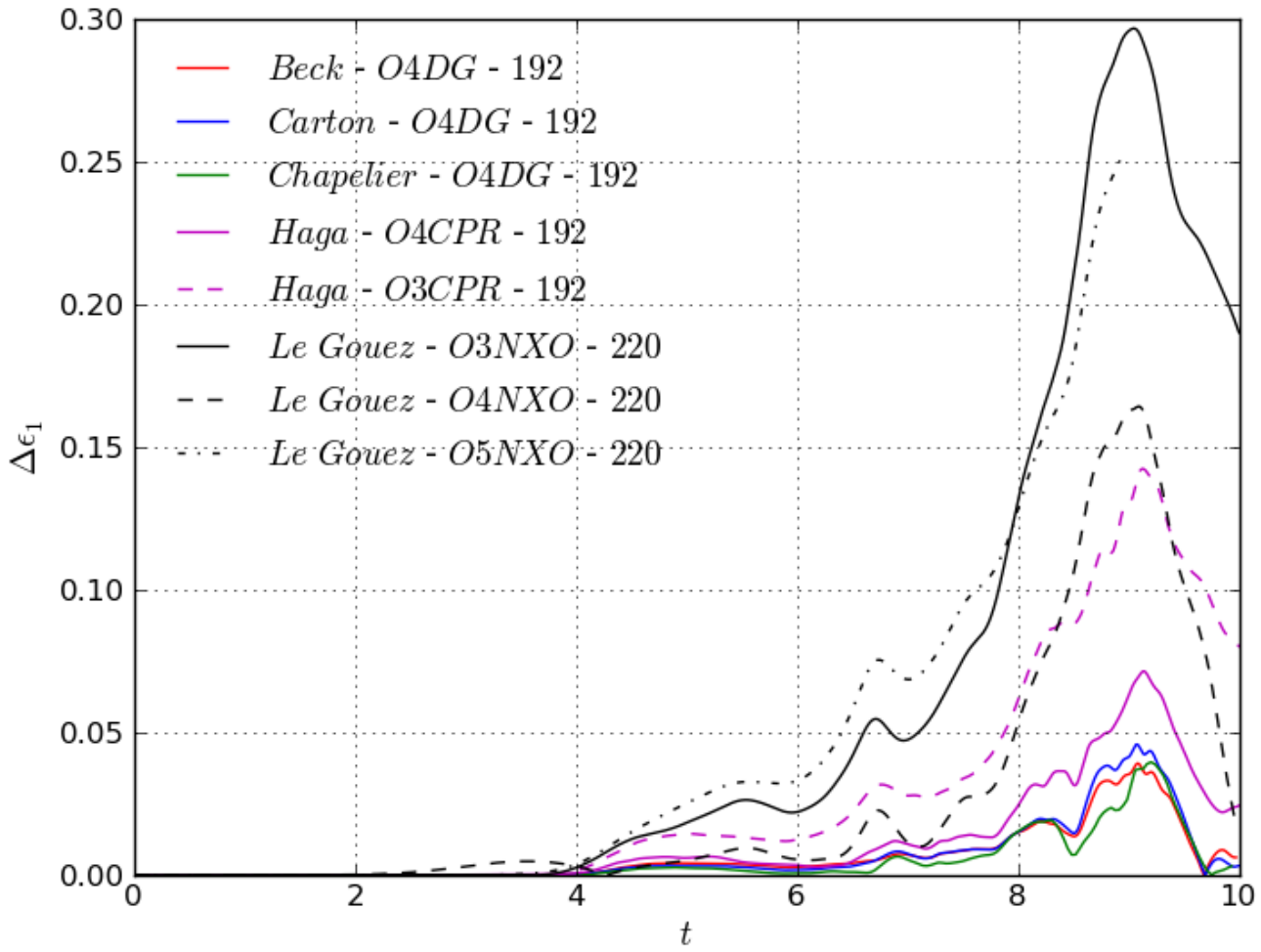


Theoretical and measured dissipation / 192

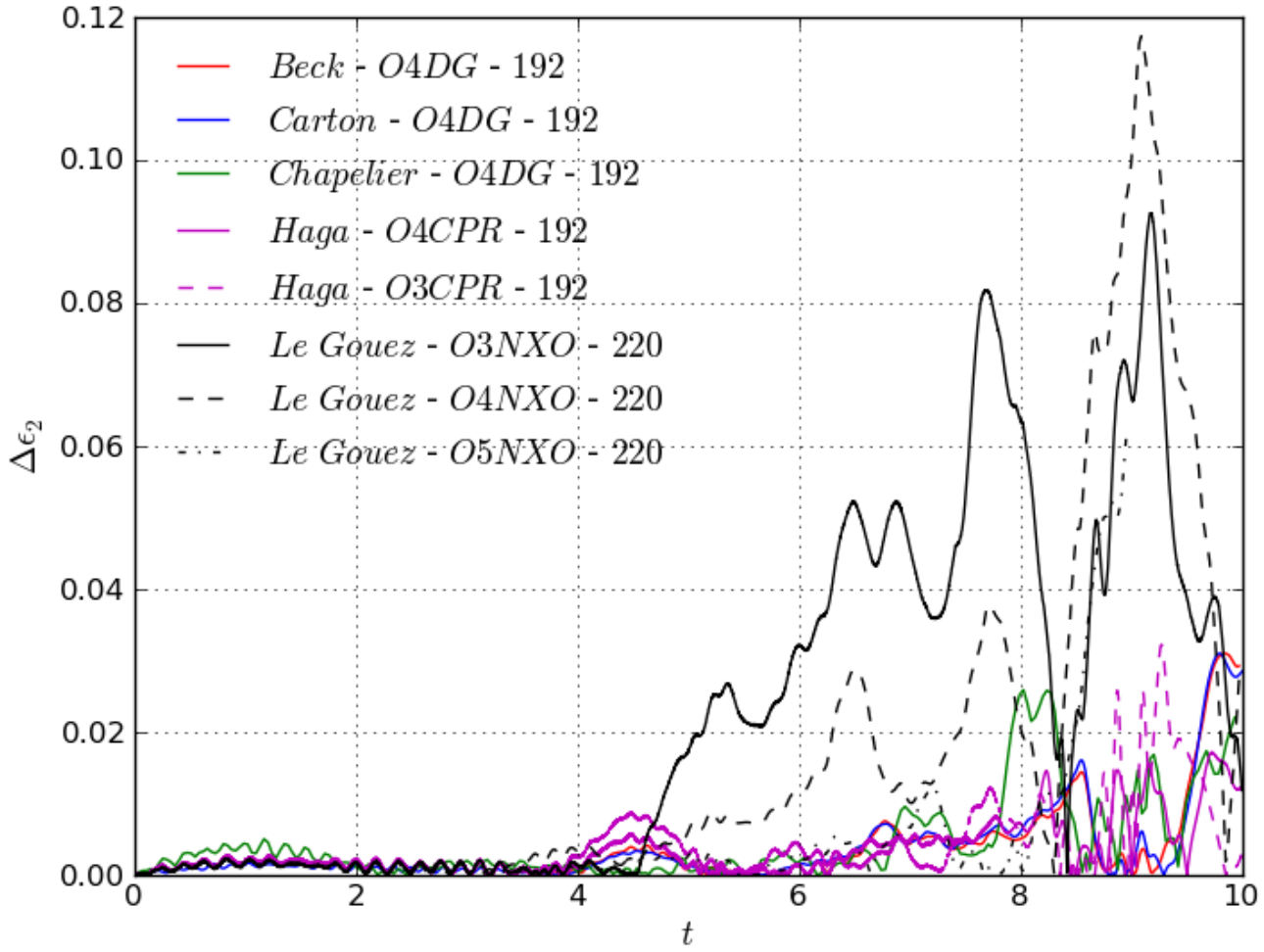




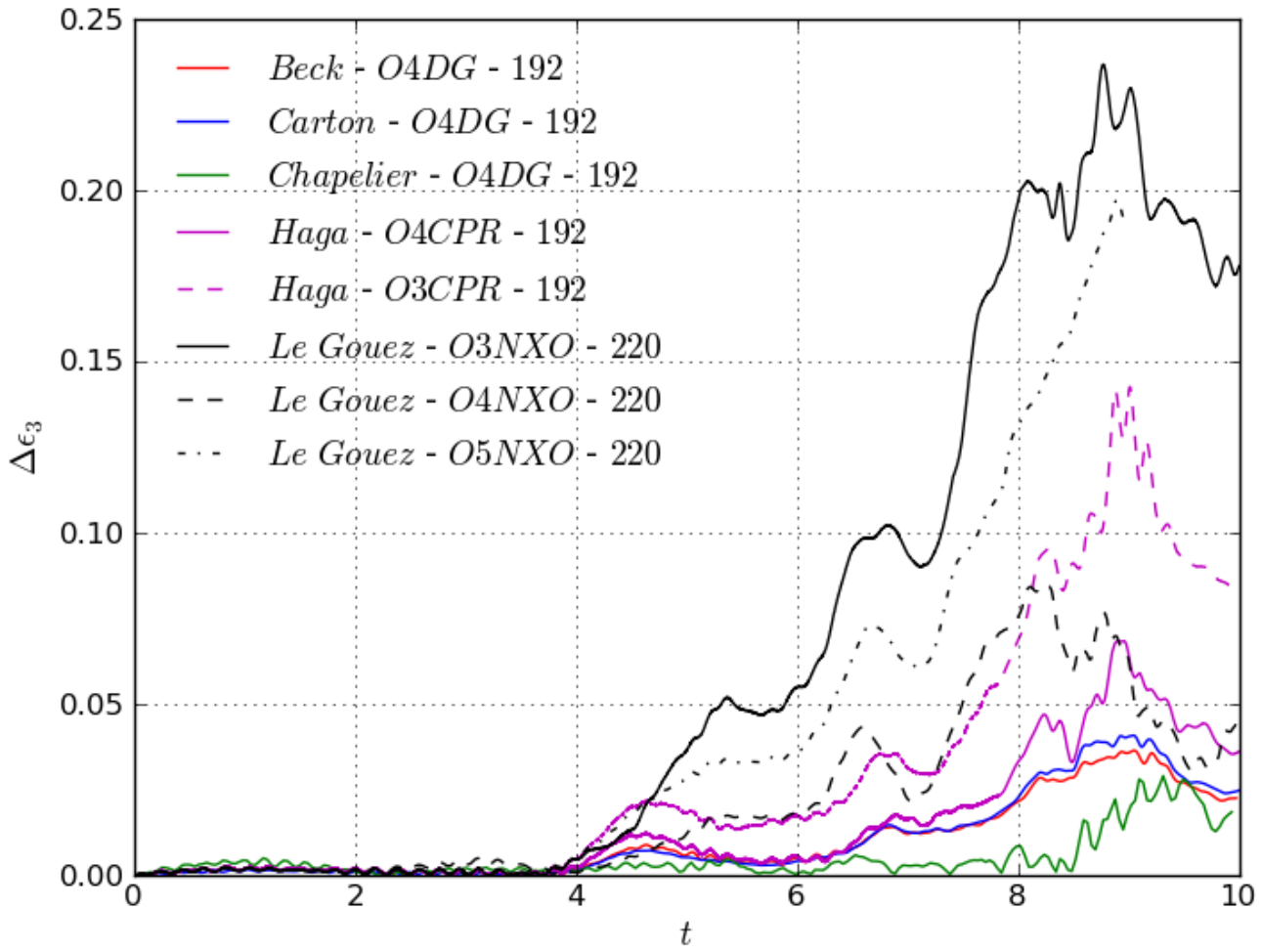
Error on theoretical dissipation / 192



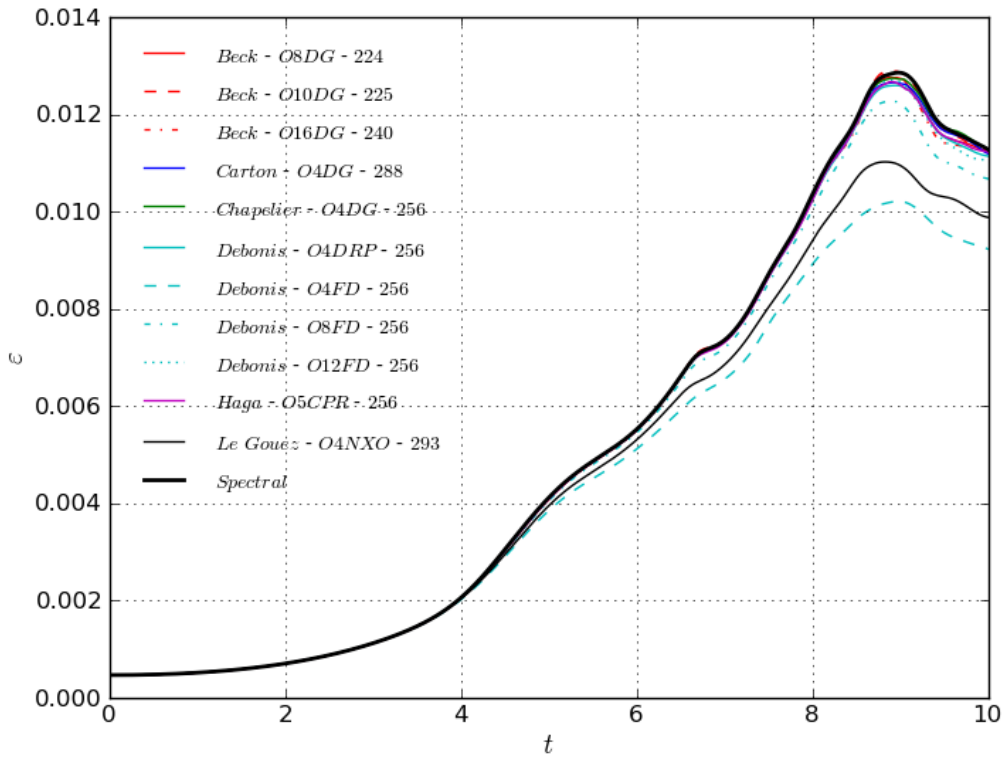
Error on measured dissipation / 192

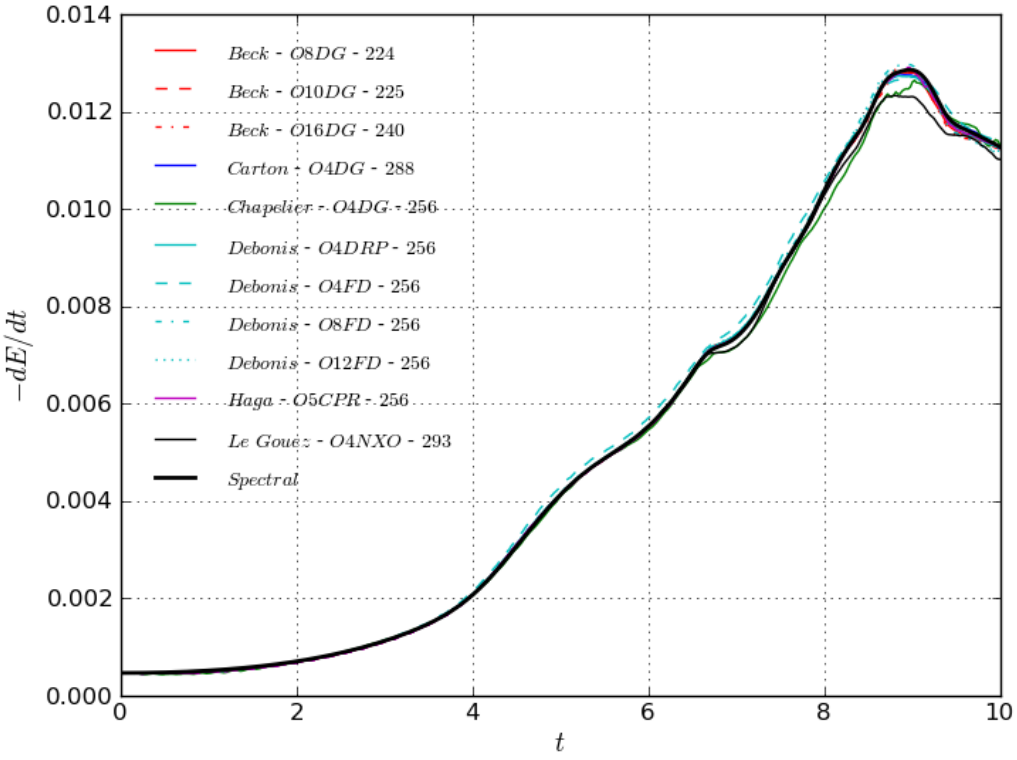


Difference theoretical & measured dissipation / 192

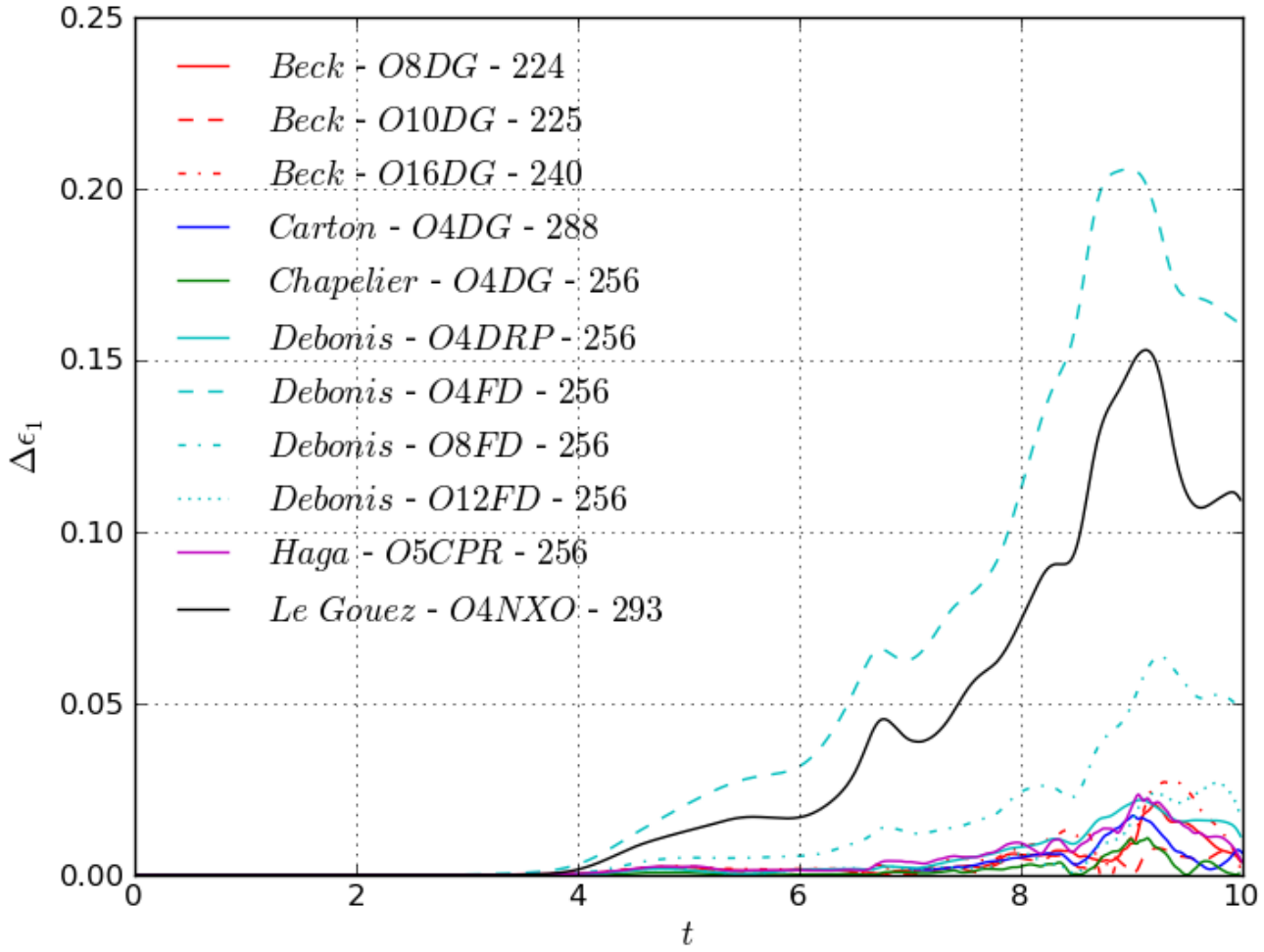


Measured vs theoretical dissipation / 256

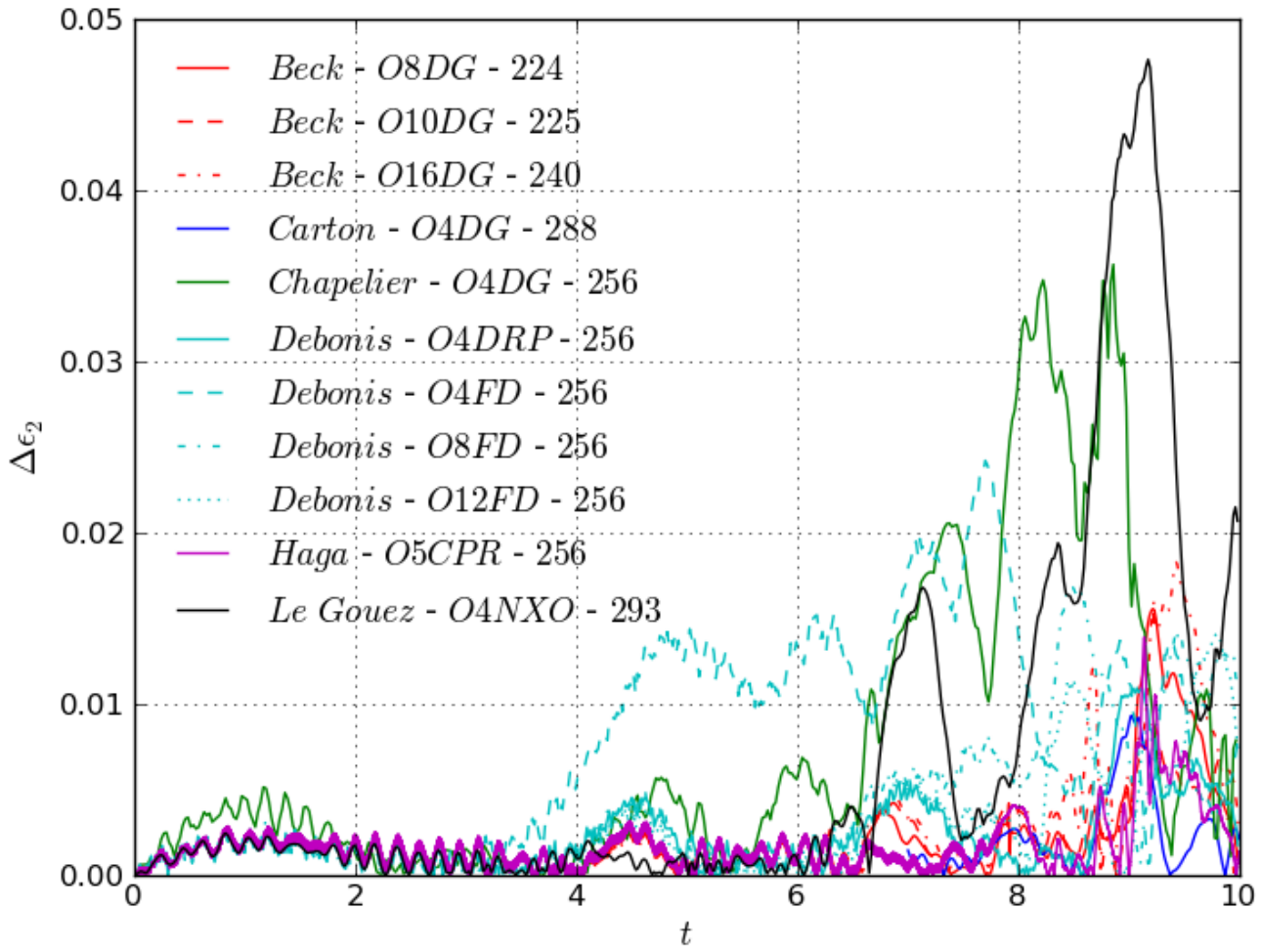




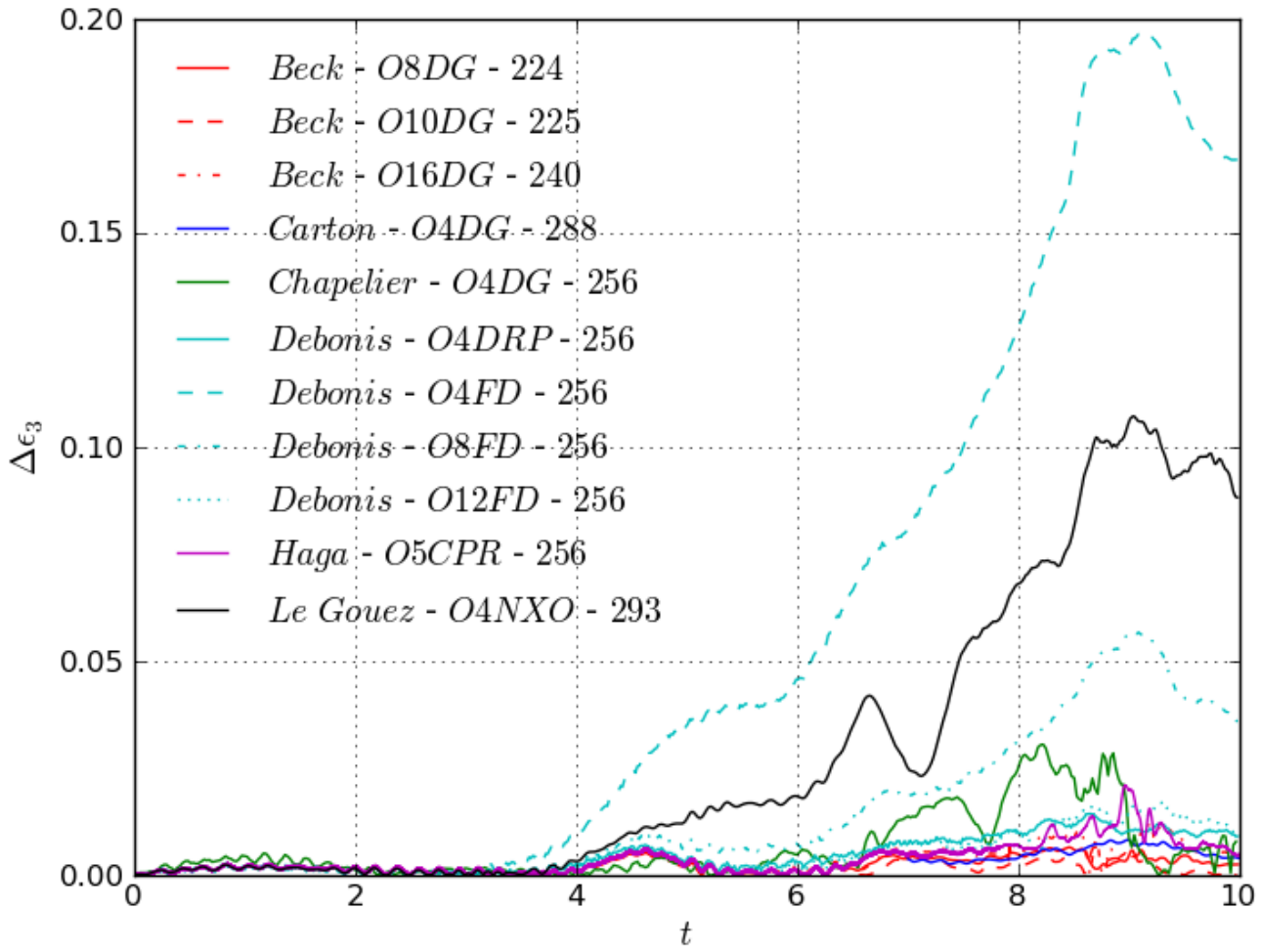
Error on theoretical dissipation / 256



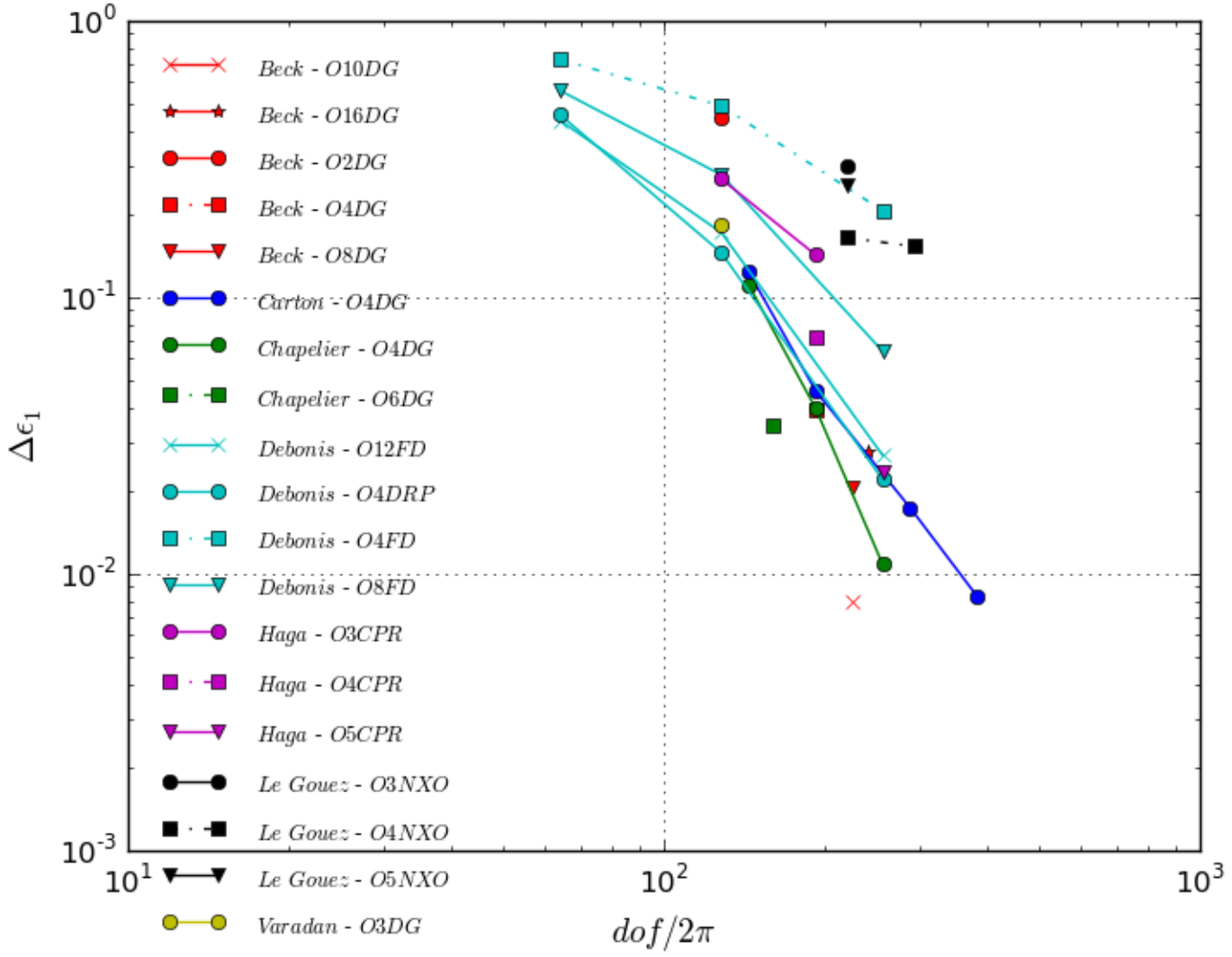
Error on measured dissipation / 256



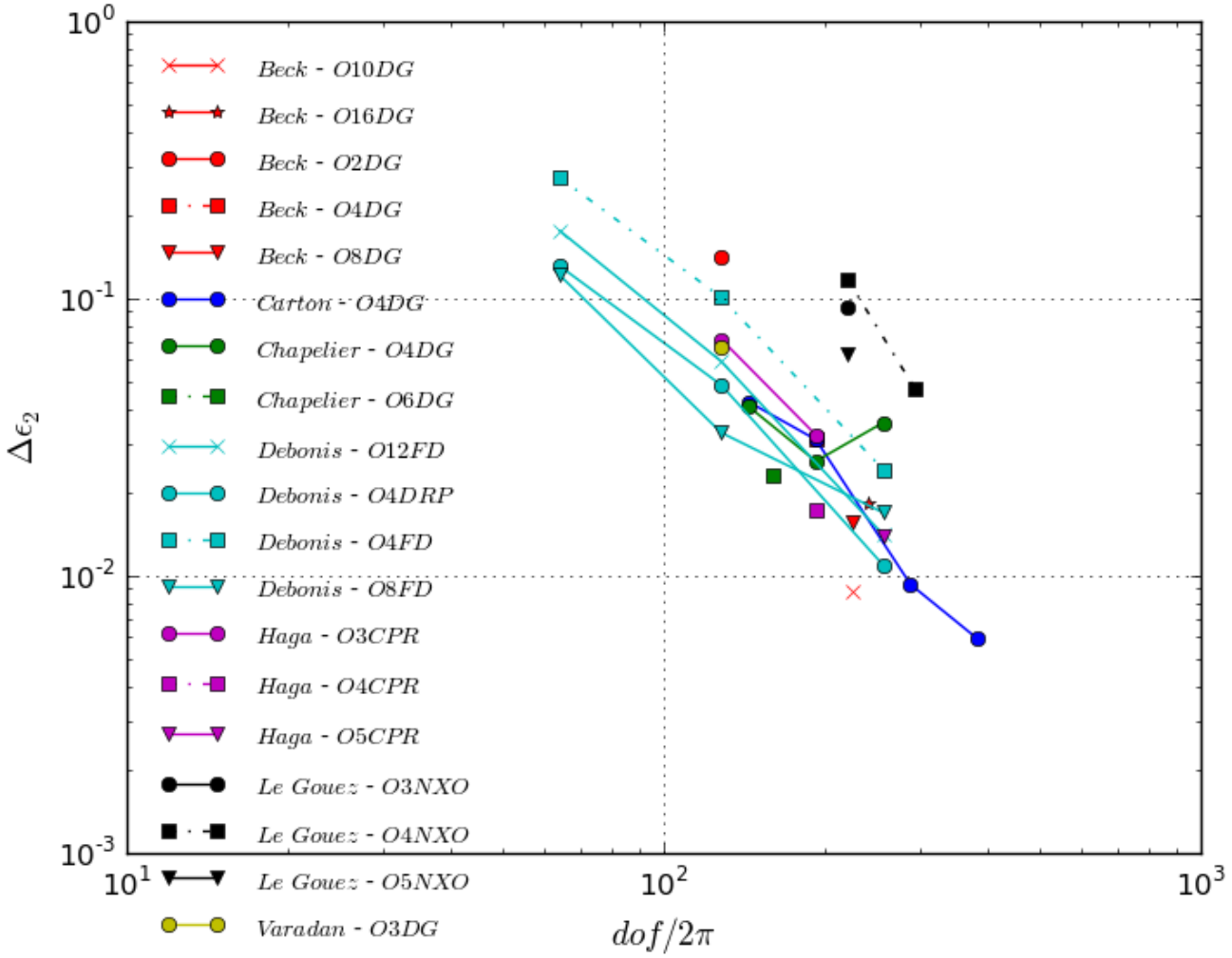
Difference theoretical & measured dissipation / 256



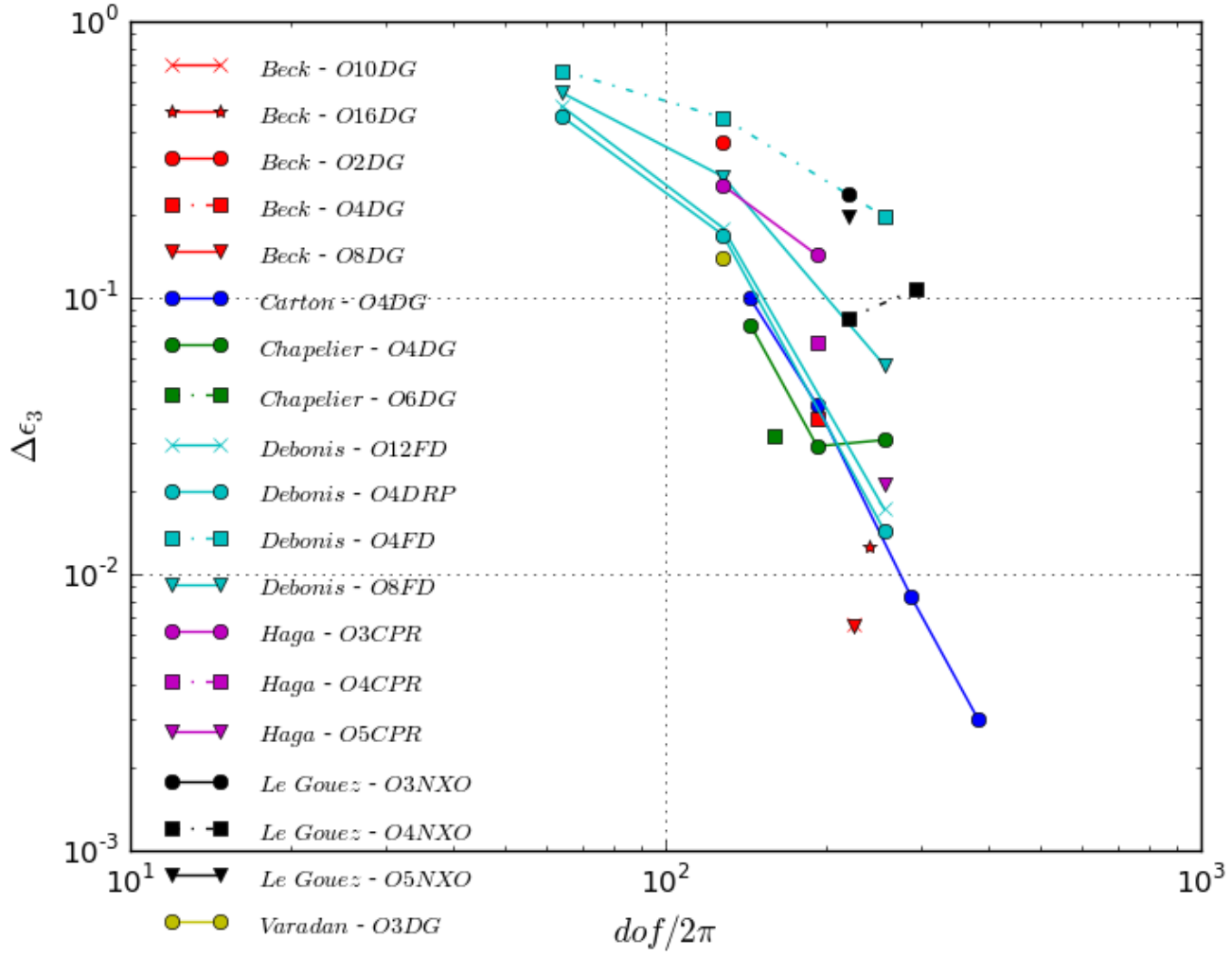
Error on theoretical dissipation vs resolution



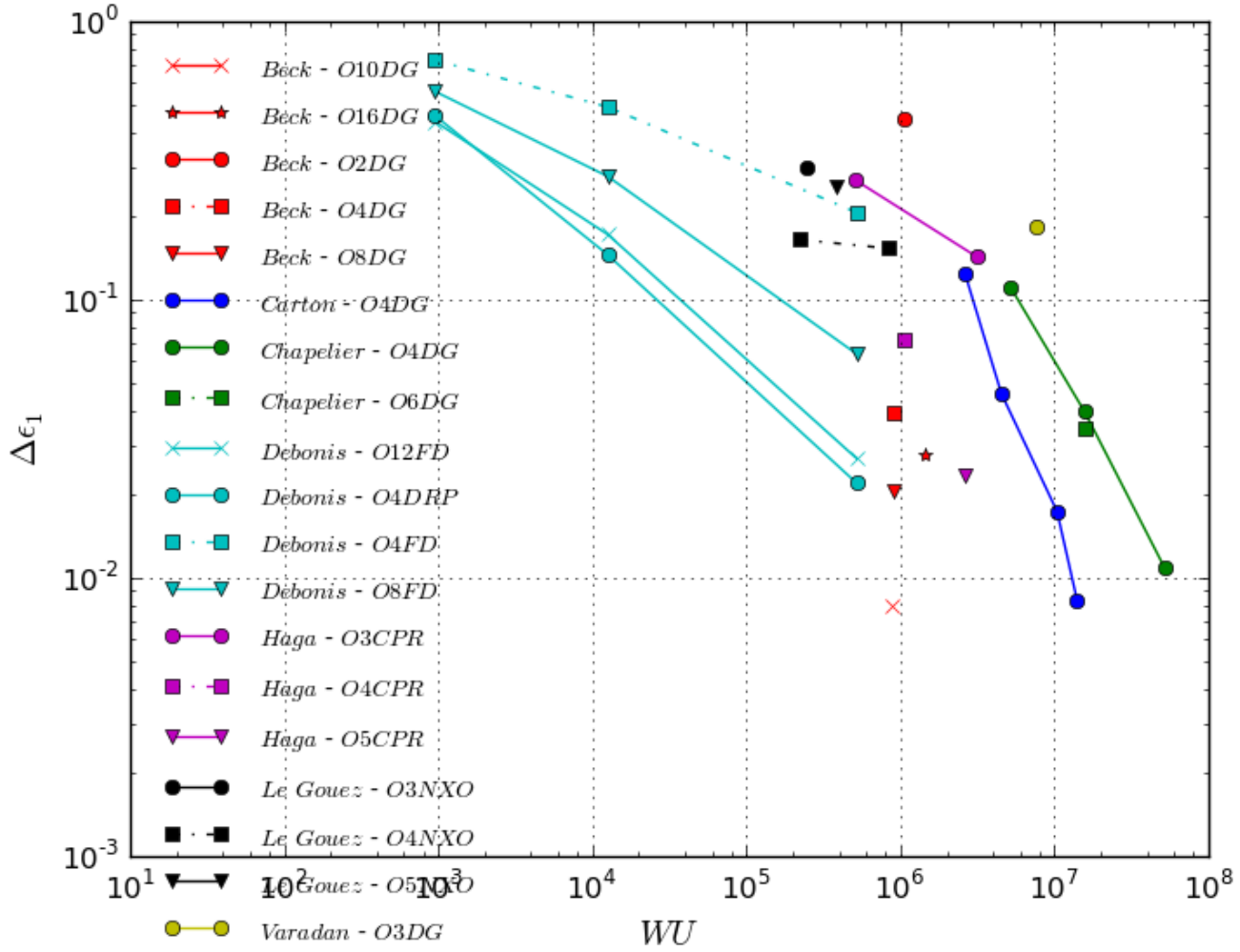
Error on measured dissipation vs resolution



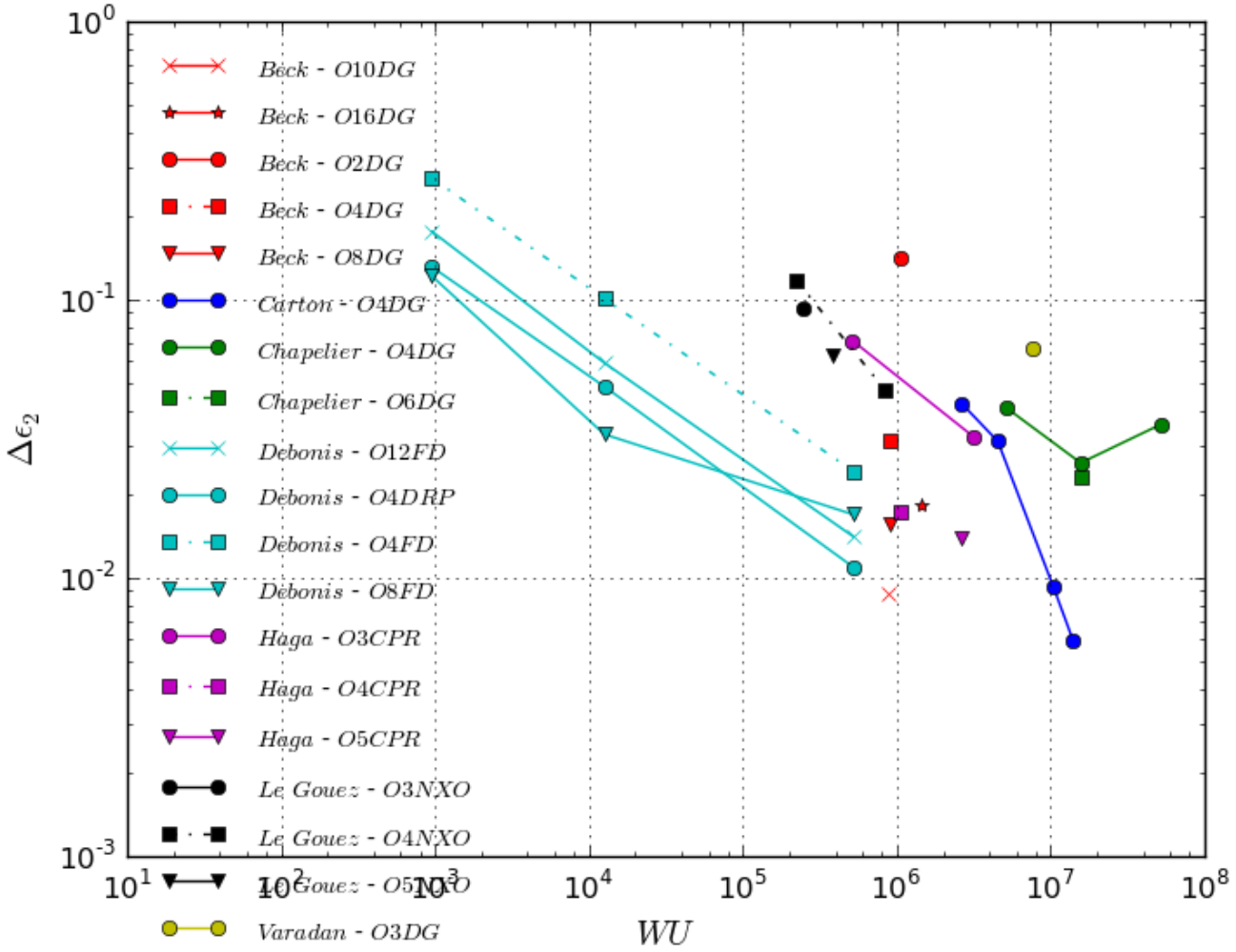
Difference enstrophy – dissipation vs resolution



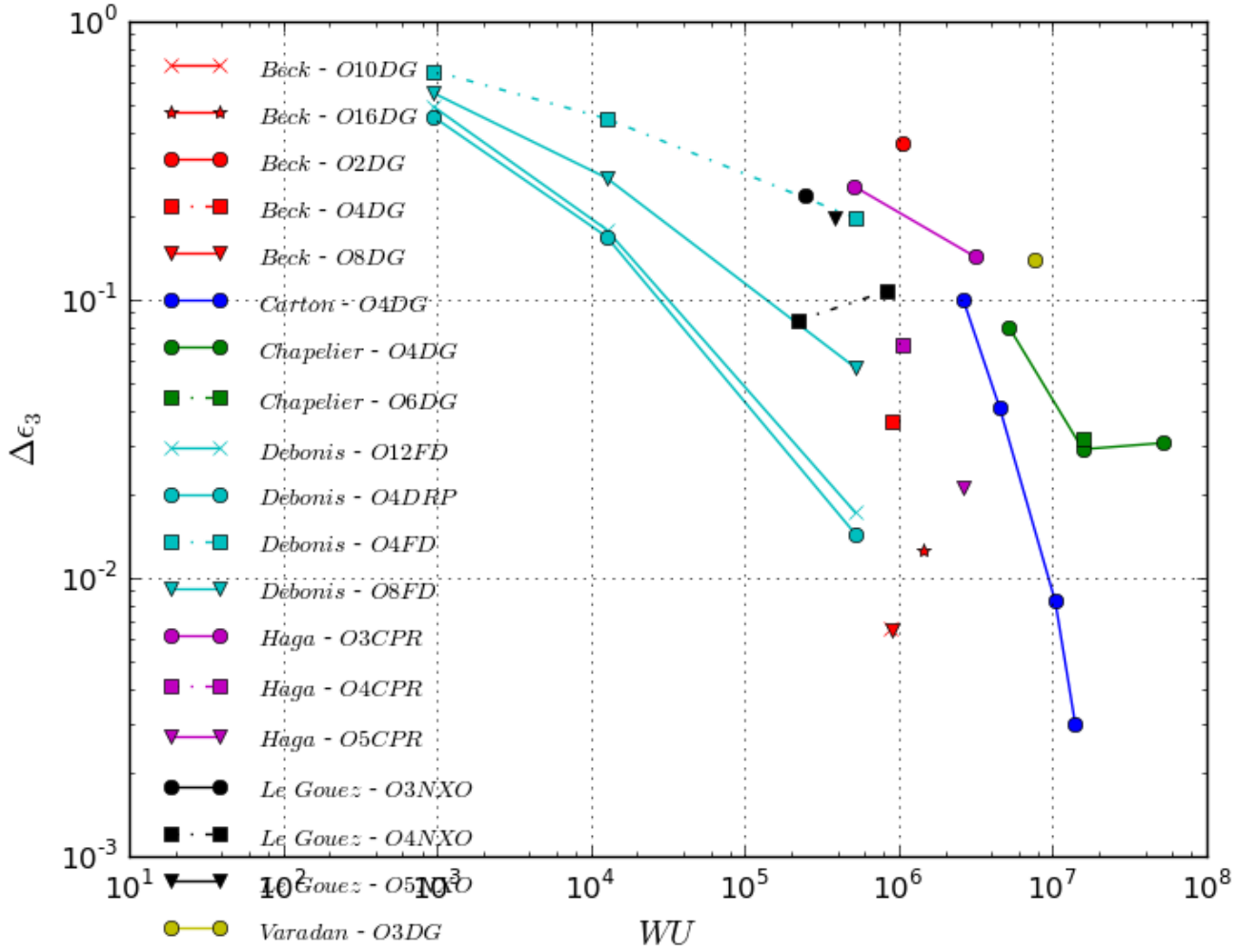
Error on theoretical dissipation vs CPU time



Error on measured dissipation vs CPU



Difference enstrophy – dissipation vs CPU



Concluding remarks

- **Conclusions**
  - **Error evolution (difference measured vs theoretical diss.)**
    - Nyquist criterion: resolution ~ 256, O ≥ 4
    - Error ~ h^{p+1}
    - Dissipation well targeted → ILES ?
    - Dispersion takes over wrt dissipation (DRP/DG)
  - **Computational efficiency**
    - Explicit : increasing order for constant dof pays
    - DG / CRP methods higher accuracy / resolution
    - FD/FV & dedicated codes : higher computational efficiency
- **Additional work**
  - **Error based on theoretical vs measured dissipation**
    - Independent of reference solution !
    - Qualitative correlation with error measure should be quantified
  - **Homogenize test matrices**
    - Grid convergence constant order
    - Order convergence on resolution ~ 256
  - **Resolve anomalies in some vorticity plots**

Acknowledgments

- **Reference results and case setup**
  - **M. Duponcheel (UCL/FTL)**
  - **G. Winckelmans (UCL/FTL)**