Computation of Fictitious Gas Flow with Euler Equations

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Abstract

The Fictitious Gas Concept supports some computational design methods to construct shock-free transonic flows. It was originally developed for potential flows, here it is introduced to the Euler equations for more general applications. A new equation of state needs to be defined in order to simulate results of the simpler potential approach. An operational numerical Euler code was chosen for the modifications and tested on a basic boundary value problem: The inviscid flow past a circular cylinder with a local supersonic flow region. The numerical computation is based on a finite volume method to solve the time dependent Euler equations in integral form. Conclusions are drawn for a physical explanation of the hitherto abstract “fictitious” gas: an internal momentum and energy supply/removal is modelled and the results for locally nonisentropic flow may be interpreted as an internal cooling / heating process controlled by the local flow velocity.

Introduction

The Fictitious Gas Concept (FGC) and its applications to transonic design of shock-free aerodynamic configurations were proposed by Sobieczky and extensively investigated by Sobieczky and Seebass [1], [2]. The main purpose of the FGC is to design airfoil and wing shapes as well as turbomachinery components which exhibit shock - free supercritical flows at operation conditions. In a first step of the numerical solution the subsonic part of the flow is solved with an altered perfect gasdynamic law in the supersonic domain, which results in a preliminary elliptic replacement within this domain - a “fictitious” part of the flow. Because of its elliptic type, this flow is shock-free and carries conditions to support also shock-free “real” transonic flow, which in a second step and compatibly with the sonic surface found in the first step, still has to be computed. The correct mixed type structure of the transonic field is recovered in this second step: real supersonic flow calculation uses the sonic surface as initial data and yields the boundary shape wetted by supersonic flow compatible with the combined subsonic / supersonic flow domains.

During the 1980's many operational computer flow analysis programs based on potential theory have been extended to be design tools by the use of the FGC. With new inverse design tools available today for airfoils and simple wings as reviewed in [3], the value of the concept may be reduced to applications in complex configuration optimization, where use of the FG in combination with flexible direct geometry generators allows for control of complex local supersonic flow domains. It has been shown by Zhu and Sobieczky[4], that for engineering applications the abovementioned second step of computing the real supersonic domain can be replaced by a simple surface alteration within the sonic bubble, with shape parameters defined by the extent and size of this bubble resulting from a FG calculation. Nearly shock-free flow results from this approach, as an easily obtained precondition for an accelerated aerodynamic optimization strategy.

With faster computers and accelerated CFD software available, the use of Euler solvers for practical aerodynamic design becomes feasible and all techniques successfully developed with potential theory should be available now also for the use of these improved CFD methods in applied aerodynamics. In this situation we may ask the question:

How can the FGC be implemented to the Euler equations and is there an explanation from gasdynamics or flow physics supporting the concept? In this article we duplicate FG models developed for potential theory with an Euler code. Illustrations are shown for the simple but computationally non-trivial example of transonic flow past the circular cylinder. The computer code of Kroll and Rossow [5], [6], [7] is applied to solve the two-dimensional Euler equations using a finite volume spatial discretization and Runge-Kutta time stepping schemes as developed by Jameson, Schmidt and Turkel [8].

Euler equations and fictitious gasdynamic relations

The two-dimensional Euler equations describing conservation of mass, momentum and energy for unsteady inviscid flows are written in the following conservative form

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0, \qquad (1)$$

where

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho e \end{bmatrix}, \qquad F = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ (\rho e + p) u \end{bmatrix}, \qquad G = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ (\rho e + p) v \end{bmatrix} . \qquad (2)$$

For the perfect gas the equation of state can be described as

$$p = (\kappa - 1)\rho\left(e - \frac{u^2 + v^2}{2}\right) . \qquad (3)$$

For the fictitious gas gasdynamic relations are asked for to provide a change from a locally hyperbolic to an elliptic or parabolic behavior. To acquire this an analytical fictitious relation for the speed of sound

$$A_f(Q) \geq Q \qquad (4)$$

needs to be prescribed, which gives with a relation for the compatible equation of state

$$A_f^2 = \frac{1}{\gamma} \cdot \frac{dP_f}{dD_f} . \qquad (5)$$

Here the non-dimensional variables are defined by sonic (critical) conditions of ideal gas flow :

$$A = \frac{a}{a*}, \qquad Q = \frac{q}{a*}, \qquad D = \frac{\rho}{\rho*}, \qquad P = \frac{p}{p*} .$$

Model functions for fictitious speed of sound and the fictitious density implemented respectively into various non-conservative and conservative potential codes are

$$A_f^2 = Q\left[1 + \frac{1}{\lambda}(Q - 1)\right], \qquad (6)$$

$$D_f = \left[1 + \frac{1}{\lambda}(Q-1)\right]^{-\lambda} . \qquad (7)$$

With equations (4)-(6) the relation for fictitious pressure,to be implemented into the Euler equations, can be obtained

$$P_f = 1 + \frac{\lambda\gamma}{2-\lambda}\left[2 - \lambda D^{\left(\frac{\lambda-2}{\lambda}\right)} - (2-\lambda)D^{\left(\frac{\lambda-1}{\lambda}\right)}\right], \qquad (8)$$

or

$$P_f = 1 + \frac{2\lambda\gamma}{2-\lambda} - \frac{\lambda\gamma}{2-\lambda}(Q+1)\left[1 + \frac{1}{\lambda}(Q-1)\right]^{1-\lambda} . \qquad (9)$$

These relations constitute a one-parametric ($0 < \lambda \leq 1$) family of fictitious gases which includes parabolic sonic flow ($\lambda = 1$) and approaches an incompressible limit for the fictitious flow ($\lambda \rightarrow 0$), viz. $\rho = \rho^*$.

In the present computation the fictitious pressure-density relation (8) is used as an equation of state within isolated supercritical domains where $Q \geq 1$. At the sonic line ($Q = 1$) the real subsonic flow and the fictitious flow have a smooth continuation. Even though the total energy (e) is independent of the fictitious pressure, the total enthalpy ($h = e + p/\rho$) as well as the entropy ($s = p/\rho^\gamma$) in fictitious gas flow deviates from that in real flow. By contrast to the real subsonic isentropic flow the fictitious gas flow is a non-isentropic one. The fictitious density-velocity relation (7) will be proven as a numerical result of the manipulated equations.

Discussion of fictitious gas models

The Euler equations for the FG flow can be transferred into another form. Assuming that p at the left hand side of Eq. (1) is a perfect gas pressure, the RHS for the FG model has then the following form:

$$RHS = \begin{bmatrix} 0 \\ \frac{\partial}{\partial x}(\Delta p) \\ \frac{\partial}{\partial y}(\Delta p) \\ \frac{\partial}{\partial x}(\Delta p \cdot u) + \frac{\partial}{\partial y}(\Delta p \cdot v) \end{bmatrix}, \qquad (10)$$

where

$$\Delta p = p - p_f \cdot$$

p and p_f are respectively determined by Eq. (2) and Eq. (7) ($p_f = P_f \cdot p^*$). In this case the Euler equations for the FG model are equal to those for the real flow with additional source terms in the momentum and energy equations. Therefore the FG flow can be interpreted as a flow with internal momentum and energy supply/removal.

Numerical algorithm

In the numerical computation the time dependent Euler equations in integral form are discretized by the finite volume approach based on the method of Jameson et al. [8]. The physical domain is covered with a body-fitted grid using curvilinear coordinates. In the present work an O-grid was generated by conformal mapping. In the finite volume spatial discretization a cell vertex scheme is applied, in which the flow variables are defined at the vertices of the cells and a volume weighted distribution formula of Hall [9] is used for the rate of change of the flow variables at a vertex. According to Kroll and Rossow in the case of the cell vertex formulation, a solid wall boundary condition is implemented by the projection of velocities. The treatment of the far field boundary conditions is based on the Riemann invariants for one-dimensional flow normal to the boundary.

In order to damp high frequency oscillations caused by the finite volume discretization with central averaging, artificial dissipative terms are introduced by a blend of first and third order dissipative fluxes. For the integration of the discrete equations to steady state an explicit 5-stage time stepping scheme of Runge-Kutta type is used with two evaluations of the dissipative terms. To accelerate the convergence of the solution of unsteady Euler equations to steady state, the techniques including local time stepping, enthalpy damping and residual averaging are applied in the present code extended to allow calculations with the fictitious gas model. As convergence criterion the relative rate of change of density is used and the accuracy is 10^{-5}.

Results and discussion

The Euler code has been used for various airfoils as well as for the simple test case of a circular cylinder in transonic flow. The critical Mach number for this boundary value problem without circulation has been determined by Van Dyke and Guttmann [10] as $M^* = 0.3982$. The present investigation is focused on supercritical flow at $M_\infty = 0.45$. The parameter λ in the fictitious gas laws is set to 0.8. The local Mach number contours for ideal gas flow and the O-grid around the circle are plotted in Fig. 1. In this transonic flow because of a strong shock, the flow field behind the circle is not symmetrical about the vertical axis. On the contrary the shock-free flow with fictitious gas within the sonic bubble is completely symmetrical about the vertical axis (Fig. 2). In Fig. 3 the fictitious density-velocity relation Eq. (6) is satisfactorily verified by the calculated values for density and velocity on all grid points within the sonic bubble. The isentropic corresponding relation is also depicted.

We ask now for an ideal gas flow which exhibits the real subsonic part as computed with using the fictitious gas within the sonic bubble and a compatible real supersonic shock-free flow pattern within this bubble. Here we skip the previously mentioned "second step" method of characteristics calculation and use instead the simpler optimization approach as developed by Zhu and Sobieczky [4]. The resulting flow field past the locally flattened circle is found by a final perfect gas flow analysis, it is found to be practically shock-free as shown in Fig. 4. The pressure distributions for the transonic perfect gas flow past the original circle with strong shock and the fictitious gas flow showing symmetrical distribution as well as the transonic practically shock-free flow past the locally deformed circle are compared in Fig. 5. The entropy productions along the contour of the original circle in transonic ideal gas and fictitious gas flow and of the modified circle in transonic ideal gas flow are shown in Fig. 6. In fictitious gas flow the entropy is first decreased and subsequently raised to its upstream value, which can be interpreted as heat removal (cooling) and subsequent equivalent heating within the sonic bubble. Compared with the increment of entropy caused by the strong shock in transonic flow past the original circle, the flow past the locally modified circle indicates practically no entropy increase which verifies the shock-free design of the contour, resulting here in a thickness reduction of about half a percent of the circle diameter.

Conclusions

The two-dimensional Euler equations in association with the fictitious gas laws are numerically solved by a finite volume spatial discretization and Runge-Kutta time stepping scheme. Numerical tests have shown that the fictitious pressure-density relation - a fictitious equation of state - is the appropriate function to be implemented into an Euler code in order to replace previous potential solver results which used the fictitious density-velocity relation. The latter agrees with numerical results of the manipulated equations. A simple surface alteration within the sonic bubble, resulting from an engineering design approach based on the fictitious gas concept, yields practically shock-free aerodynamic configurations. The fictitious gas flow is non-isentropic, which can be physically explained as an internal cooling/heating process controlled by the local flow velocity. The introduction of the fictitious gas model into the Euler equations provides a more general method of designing transonic flow fields with favorable aerodynamic characteristics.

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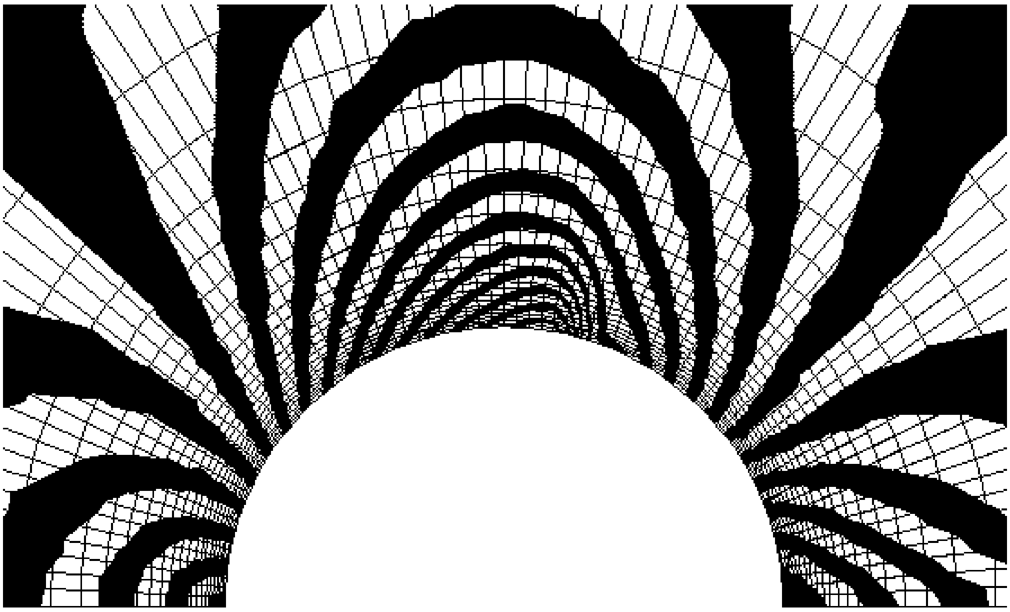

Fig. 1: Transonic perfect gas flow past circular cylinder, $M_\infty = 0.45$. Local Mach number contours and grid (O-grid with 160 x 32 points).

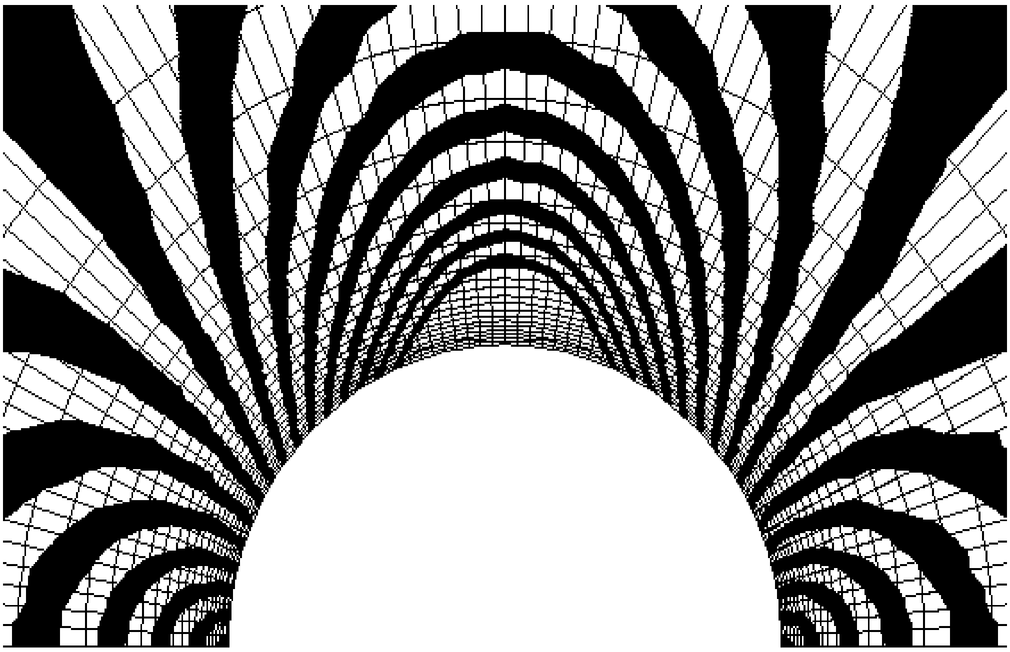

Fig. 2: Transonic flow $M_\infty = 0.45$ with fictitious gas ($\lambda = 0.8$) within sonic bubble. Local Mach number contours and grid.

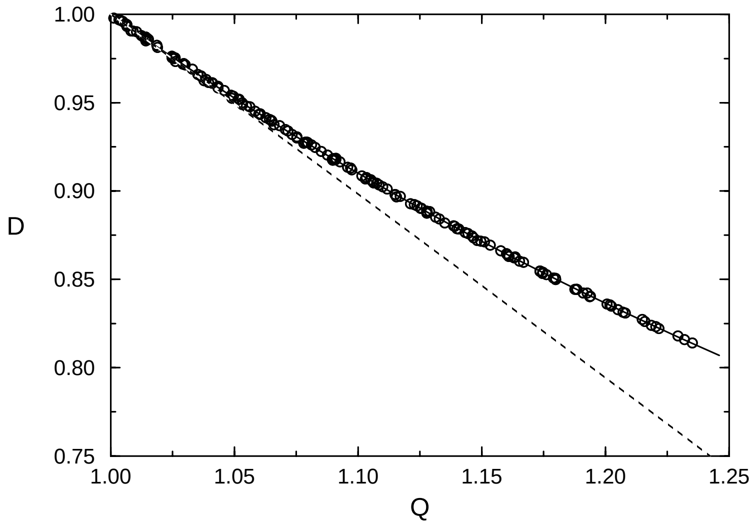

Fig. 3: Verification of fictitious density - velocity relation in numerical results for fictitious flow past circle: solid line is analytical relation Eq. (6) with $\lambda = 0.8$, symbols are local values for density on all grid points within sonic bubble, dashed line is isentropic density velocity relation for ideal gas

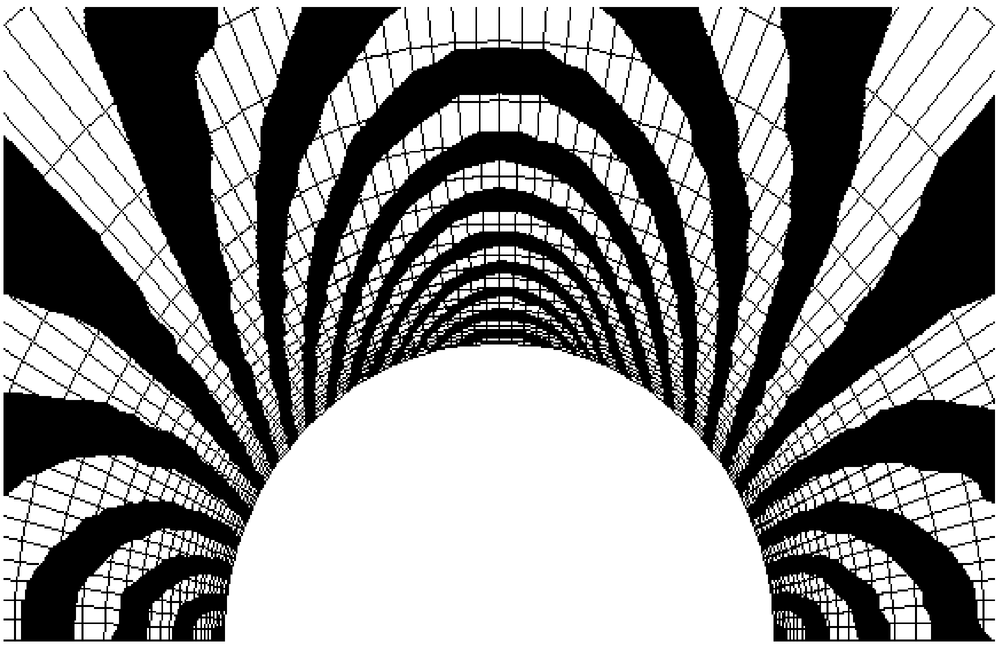

Fig. 4: Transonic perfect gas flow analysis of circle contour modified within previously found fictitious domain, $M_\infty = 0.45$. Local Mach number contours and grid.

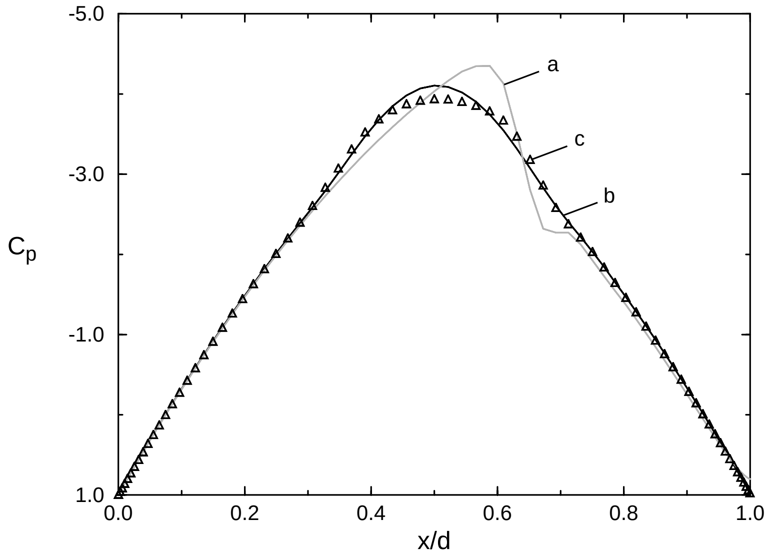

Fig. 5: Comparison of pressure distributions for (a) transonic flow past circle with strong shock, (b) fictitious gas flow past circle showing symmetrical distribution, and (c) for transonic practically shock-free flow past locally deformed circle, resulting from design method based on fictitious gas concept.

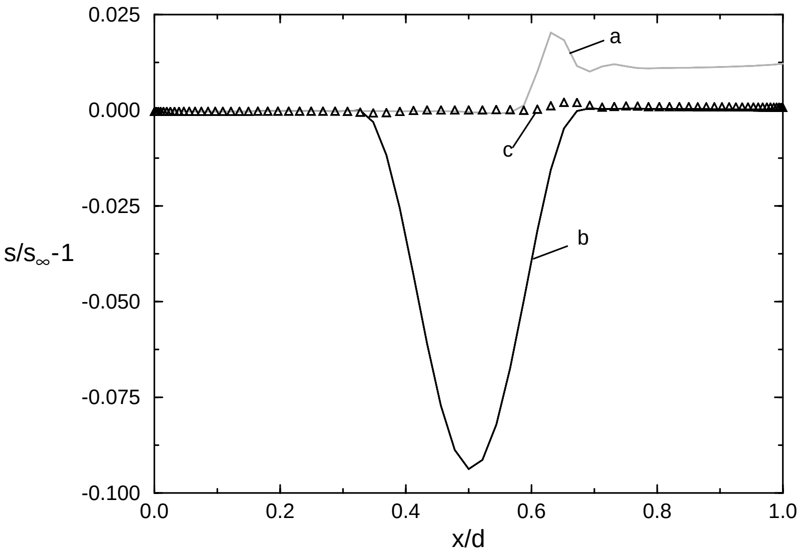

Fig. 6: Comparison of entropy production along contour of original circle in (a) transonic ideal gas and (b) fictitious gas flow and (c) of modified circle in transonic ideal gas flow