AIAA-95-1819-CP

A Design Method for Supersonic Transport Wings

Pei Li [1]
Aerospace Engineering Sciences
University of Colorado, Boulder

Helmut Sobieczky [2]
DLR German Aerospace Research Establishment
Göttingen, Germany

Richard Seebass [3]
Aerospace Engineering Sciences
University of Colorado, Boulder

Abstract
We report here results from our use of the fictitious gas method for the design of supersonic transport wings using the Euler equations. While the example chosen here is the oblique flying wing, the technique has general application to supersonic wings with subsonic leading edges.

1. Introduction

Air travel and air cargo traffic have grown, with few pauses, for twenty-five years, albeit at declining rates. International passenger traffic has grown faster than domestic passenger traffic and this has grown most rapidly on routes with and linked to the Asia Pacific region. The proportion of business travel has declined relative to leisure travel. About sixty percent of current scheduled air travel is coach and sixty percent of this is discount coach. These trends have focused recent aircraft design attention on a New Large Aircraft and on a second generation SST, described in Europe as Concorde II, and in the U.S. as an HSCT (high speed civil transport).

Market estimates for a second generation supersonic transport differ, but it is clear that a supersonic aircraft that can compete economically with subsonic transports will command a very considerable market, even if restricted to subsonic operation over land. To compete economically the fares will have to match the undiscounted fares for subsonic flight. The trends mentioned above suggest that two-thirds of the seats should be for first class and business passengers, and one-third for coach passengers. A large craft, capable of efficient near sonic operation overland and supersonic operation over water, such as the Oblique Flying Wing, would command a very substantial market.

We describe here a tool for the efficient design of supersonic wings with subsonic leading edges that has general application to supersonic transport aircraft. We illustrate the method through its application to the aerodynamic design of an oblique wing flying supersonically.

Thanks to rapidly changing computer technology replete with its robust computational tools, data bases, and informations systems, today's aircraft designer has a rich environment for design. As but one example of this environment, we note the coupled numerical solution of the Navier-Stokes and structural dynamics equations for a supersonic transport deforming under changing aerodynamic loads and thereby altering these loads. Soon to be published work already shows this advance for an HSCT wing using parallel processors (Cray T3D, Paragon XP/S, IBM SP2) [1].

The effectiveness of a designer in this environment depends on the completeness, robustness and reliability of these tools and, more importantly, on the designer's knowledge of the supporting theories and past experience. Wrong or unhelpful results are more likely to derive from incomplete understanding or lack of experience than from unreliable or incomplete tools. Consequently, we note here our own lack of experience in aerodynamic design.

In addition to the tools that allow the computation of an aerodynamic flow, an aerodynamic designer should be equipped with the knowledge of the theoretical optima, inverse techniques where desired results can be prescribed, and the capabilities of optimization routines. Limited reviews of the capabilities of inverse design and optimization techniques may be found in [2] and [3].

The designer of a supersonic transport wing, using commonly available tools, would probably fail to find the linear optimum. The linear optimum for lift was given in 1952 by R.T. Jones [4], namely a wing with an elliptic loading swept behind the Mach cone flying obliquely. Only an optimization tool that supported and encouraged the exploration of asymmetrical solutions would discover such a design. Theoretical knowledge alone will not suffice. It led Kogan [5] [6] to note that the optimum design

1. Research Associate
2. Senior Research Scientist
3. Professor, Visitor, DLR, Göttingen

with subsonic leading edges had an elliptic loading, but Kogan dismissed the utility of subsonic leading edges for supersonic aircraft.

The efficiency of turbojet transport aircraft is proportional to $M_\infty L/D$, where these symbols have their usual meaning of freestream Mach number and wing (or airfoil) lift and drag. This derives from the nearly linear increase in the propulsive efficiency of turbojet propulsion with speed or Mach number. When a subsonic freestream Mach number exceeds the critical Mach number, the wings develop extended supersonic regions normally terminated by a shock wave. This shock wave thickens the boundary layer on the upper surface, reducing the shed vorticity and the wing's overall circulation and, consequently, its lift.

Investigators, first through experiment [7] [8], and then theory [9] [10], noted that there were isolated airfoil shapes that, for a given freestream Mach number and lift, were without, or nearly without, shock waves. These airfoils came to be termed "supercritical" airfoils in the sense that they evidenced a high efficiency at supercritical Mach numbers. Perhaps a more appropriate description is "shock-free" airfoils (and for that matter "shock-free" wings) in the sense that in a neighborhood of their maximum $M_\infty L/D$, at some slightly lower $M_\infty L/D$, they are shock-free. At this shock-free point their drag as a function of Mach number for fixed lift is a minimum (for supercritical Mach numbers). It is for this reason that these designs evidence high $M_\infty L/D$ (see e.g., Figure 4).

Because of their higher efficiency, varied but remarkable theoretical tools were developed to generate these shock-free airfoils. A brief review of these design tools is provided in [11]. Among these tools only one, the fictitious gas method of Sobieczky [12], provides a method that may also be used for three-dimensional flows [13] and, remarkably for supersonic flows as well [31]. Knowledge derived from this tool provided the necessary insight for optimization routines to be used to generate shock-free airfoils and wings [15]. It was also shown to be easy to incorporate boundary layer displacement effects in these designs [16]. The further development of this tool by industry has provided higher efficiencies for current subsonic transports [17][18][19].

2. Post World War II Supersonic Wing Design

Busemann's remarkable discovery of the virtues of wing sweep for supercritical and supersonic Mach numbers [20], which Ludwieg soon verified experimentally, see e. g. [21], and Jones' rediscovery [22] and experimental reverification [23] a decade later, made it clear that efficient aerodynamic designs at these high speeds were possible. This soon led to supersonic military aircraft and the contemplation, in the early 1950s by the British aeronautical establishment, of a supersonic transport.

Early discussion focused on the virtues and vices of forward and rearward wing sweep. R.T. Jones recently commented that the oblique wing was humorously suggested at the time as a compromise between the advocates of forward and rearward sweep. Forward sweep promised more lift for the same size wing, less induced drag, increased aileron effectiveness, better control in stall, and the same high speed performance. These observations had led earlier to the Junkers 287 jet bomber and its modestly swept-forward wings. But such wings are structurally unstable in the sense that the lift developed increases the outboard angle of attack leading to a further increase in load. Overcoming this instability imposes weight penalty due to the increased wing stiffness needed. Swept back wings are structurally stable. Sweepback also provides additional directional and lateral stability (sometimes so much that negative dihederal is needed to offset this stability).

As consideration of a supersonic transport progressed in Britain, various configurations were studied. These included: an M-wing with the inboard portions of the wing swept forward to their juncture with the engines, then swept back; a thin wing with supersonic leading edges; an oblique flying wing; and a delta wing [24]. The principle difficulty with the delta wing was insufficient lift at low speeds. It was then discovered that nonlinear vortex lift could be derived from flow separation at the leading edge [25] [26]. Initially there was concern that this was an uncontrolled phenomenon that could not be relied upon. In support of the delta wing program, Handley Page Ltd. quickly built and flew a highly swept delta wing configuration, the HP 115. This, the first aircraft to intentionally fly with continuously separated airflow, demonstrated that such aircraft could be flown, although special care would be required on landing.

In retrospect the selection of a highly swept delta wing and its modification to the ogee planform of the Concorde seems a wise choice. The M-wing had few virtues and many vices. The thin wing with supersonic leading edges would have been difficult structurally; a recent patent, however, suggests it would have benefited from sizeable portions of laminar flow [27]. And the oblique flying wing of Figure 1 as described by G.H. Lee [28], while efficient, may have been too hard to control with 1960s technology and too large for the air travel market of the 1970s.

As the Concorde has demonstrated, flying a highly swept ogee wing with continuously separated flow at low speeds can be well handled through automated landing [24] [29]. At its design Mach number the Concorde is reasonably efficient; its principal inefficiencies occur at lower speeds.

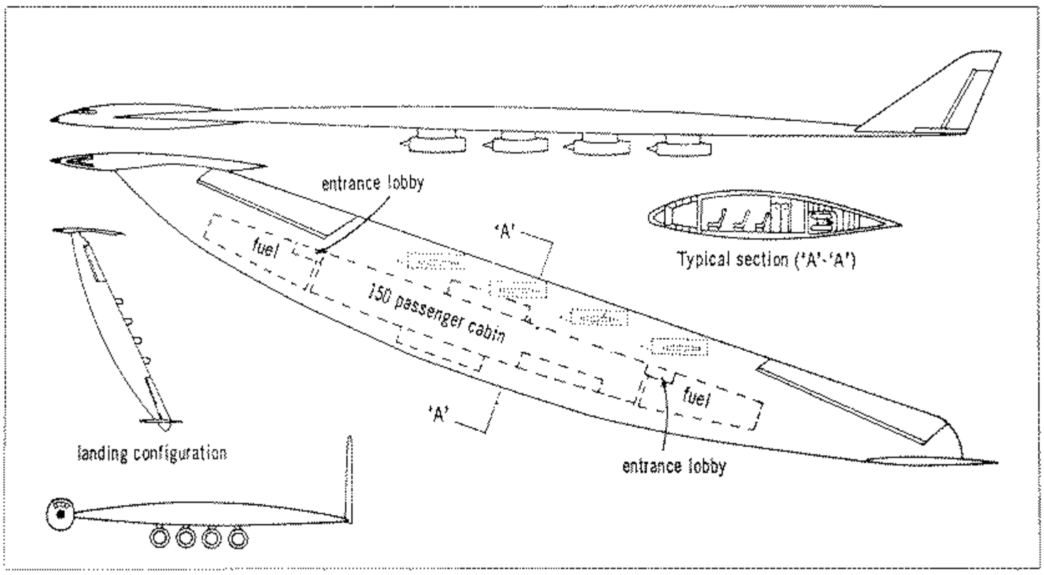

Figure 1. Mach 2 supersonic airliner proposed in 1961 by G.H. Lee of Handley Page, Ltd. [24].

3. Supersonic Wing Design

The wing of a second generation supersonic transport will most likely have its leading edge(s) swept behind the Mach cone. Higher speeds mean higher sweep angles and less efficient subsonic aerodynamics unless the aircraft has variable geometry. At cruise conditions the flow approaching the wing is that behind the shock waves emanating from the fuselage, the wing tip (forward sweep) or wing fuselage juncture (rearward sweep), or leading tip for an oblique all-wing craft. For appropriately swept wings the component of this flow normal to the wing's leading edge will be subsonic. This component accelerates over the wing to become locally "supersonic." The return of this component to subsonic flow is normally through a shock wave, just as it is on supercritical but not shock-free airfoils. This compression adversely affects the boundary layer and thereby the wing's lift and drag (see, e.g., [30]).

We can fix our ideas for supersonic flow by considering supersonic conical flow past a wing with subsonic leading edges and conical camber. Such a wing will, unless designed using special tools, have a cross-flow shock wave like that depicted in Figure 2.

While this flow is supersonic, the cross-flow plane equations are mixed, being hyperbolic outside the conical shock wave and inside the local "supersonic" cross-flow region, but subsonic elsewhere. The fictitious gas method that has proved successful for supercritical airfoils and wings applies here as well, [14], [31]. We first describe the method here, not in this context of supersonic flows such as this conical flow, but in the context of subsonic supercritical flows. When shock free these flows will normally be potential flows (outside the boundary layer). The flows we eventually consider will normally be supersonic and not necessarily irrotational. Our implementation for these flows requires the Euler equations.

Assume that we have a way to change the equations we use inside a local supersonic region so that they remain elliptic there. We can, for example, prescribe a new gas law or a new energy equation for this purpose. If we then compute the flow switching to these fictitious equations when the local flow is "supersonic," but using the correct equations elsewhere, we will generate a smooth "sonic surface" because the equations we solve are elliptic or elliptic-like. Data on this surface can then be used to recompute the "supersonic flow" with the correct equations. This recomputation will result in a new airfoil or wing surface definition that is consistent with this smooth flow, or we will find limit surfaces indicating a smooth flow does not exist. It is the art of the designer to choose the initial wing shape, and the fictitious equations, so that a smooth flow will result, thereby allowing for its redesign as a better airfoil or wing.

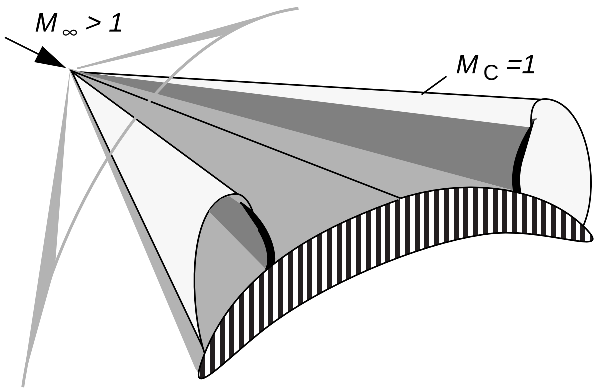

Figure 2. Embedded shock wave in a conical cross-flow.

We remind the reader of the simplicity of this method by considering first irrotational flow governed by a single equation for the potential, ϕ,

$$\nabla\cdot[\rho\nabla(\phi)] = 0,$$

where

$$(\rho/\rho_*)^{\gamma-1} = (\gamma\text{-}1)[H - (\nabla\phi)^2/2]/a_*^2.$$

Here H is the total enthalpy and a the sound speed; $()_*$ refers to sonic conditions. The simple artifice of setting the density equal its sonic value $(\rho = \rho_*)$ when the flow speed becomes equal to the sound speed ensures elliptic behaviour [12]. Other choices for the dependence of the density on the flow speed may be better. But any choice that ensures an increasing ρq will suffice to make these equations elliptic [32]. Thus the choice

$$\rho/\rho_* = (a_*/q)^P \text{ with } P < 1 \text{ and } q \geq a_*,$$

for example, provides the designer a range of elliptic solutions ($0 \leq P < 1$) that have an embedded supersonic ($q > a_*$) but elliptic domain. Any of these can be used to recompute the supersonic domain correctly and redefine thereby the airfoil or wing shape.

The calculation of the correct replacement body and its corresponding supersonic flow, while routine in two dimensions, is complex in three dimensions [33]. For low aspect ratio wings it is unreliable because of numerical instabilities. We know, however, that the real flow will everywhere have a mass flow that is lower than the fictitious flow. Reliable ways of estimating this change in mass flow (from the fictitious to real flow) have been developed by Zhu and Sobieczky [34]. They allow the redefinition of the airfoil or wing without the recomputation of the supersonic region. Since all designs should be verified by a recomputation of the flow field past the modified shape, and because of the complex numerical algorithm needed for the recalculation of the three-dimensional supersonic domain, we rely here on this simple and robust tool.

For this study we write the steady Euler equations as:

$$\nabla\cdot(\rho\boldsymbol{q}) = 0,$$

$$\boldsymbol{q}\cdot\nabla\boldsymbol{q} + \nabla p/\rho = F,$$

and

$$p/(\gamma\text{-}1)\rho + p/\rho + q^2/2 = H - \Gamma,$$

where F represents a conservative force and Γ the work done by this force that together ensure the equations remain elliptic. Or, if one prefers, a “fictitious gas” is utilized to provide a desired degree of ellipticity.

In a previous study, Li and Sobieczky [35] have shown that simple modifications can be made to the pressure and density flow speed relations to achieve this end. A single parameter ranging from 1 for a flow which is everywhere parabolic when the flow speed is equal to, or greater than, the critical speed, to 0 for a flow with its density frozen at the critical density. This change corresponds to the local addition and removal of both momentum and energy in the supersonic flow. With this artifice, we have studied two-dimensional flows and modified a circular cylinder to be shock-free at a freestream Mach number of 0.45 where there is a sizeable supersonic region.

Other possibilities exist. One can set $F = 0$ and remove energy $(\Gamma > 0)$ and then restore it to the flow as a function of $q - a_*$ for $q > a_*$. This can also be interpreted as a change in the introduction of a fictitious internal energy, e_f,

$$e_f = p[1 + \Gamma(Q)]/\rho(\gamma\text{-}1)$$

where $Q = q/a_*$. For this study we used

$$\Gamma = \lambda(Q - Q_s)^2,$$

with λ is a constant and Q_s is the value of Q at which we wish to introduce elliptic-like behaviour.

The pressure - density relation from our computation of a M_∞= 0.707 flow corresponding to λ = 3.2 is shown in Figure 3. The upper left inset corresponds to the ideal gas flow. That in the lower right inset corresponds to flow that results with the change to the fictitious internal energy for $q > a_*$ to achieve elliptic behaviour inside the sonic line. The shock in the ideal gas flow is evidenced by the saddle point in the isotachs.

In the next section we apply this method of airfoil and wing design to a supercritical airfoil useful for an oblique flying wing. We utilize the CFL3D algorithm from NASA Langley for our studies (see [36] and [37]).

4. Initial Application: Oblique Flying Wing Design

The method outlined above is, perhaps, most profitably applied to nearly conical wings, to wings flying obliquely, and to arrow wings. For our studies we have chosen an oblique flying wing (OFW). Elliptically loaded and with a Sears-Haack volume distribution, this wing provides the linear theory optimum for supersonic L/D for a given volume [38]. How well such a actually wing performs, however, goes beyond linear theory.

If we consider a finite swept wing in a supersonic flow,

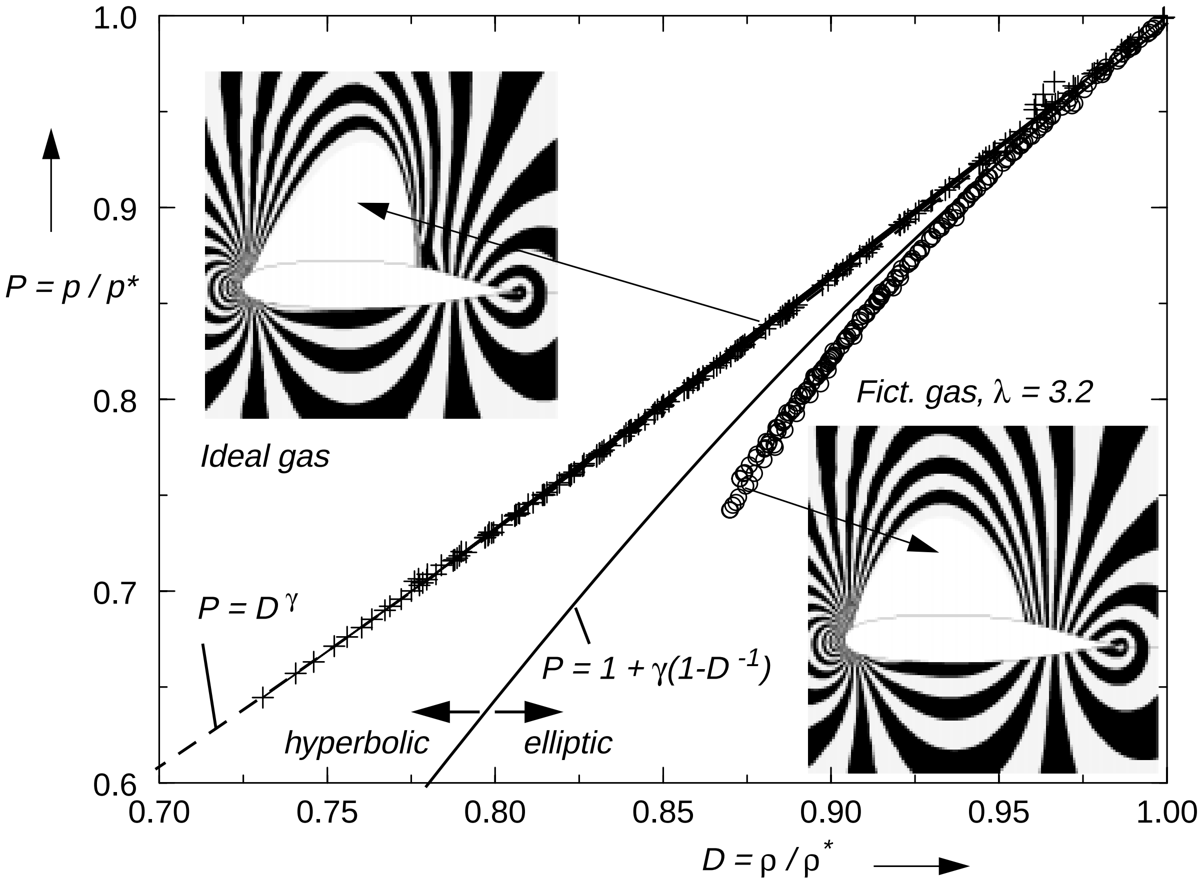

Figure 3. Pressure - density relations for ideal gas and fictitious gas calculations. Numerical evaluation of all grid points within the sonic bubble of ideal and fictitious gas inviscid airfoil flows ($M_\infty = 0.707$, $c_l = 0.6$).

then, as the span of the wing increases to infinity, the shock wave from the leading tip recedes to infinity, leaving a subsonic normal flow. Thus, for an swept wing of infinite span, we need only design a supercritcal airfoil. We then use this airfoil to create a wing of finite span and reapply the method to the design of this wing. Finally we consider the oblique supersonic flow over this wing.

An oblique flying wing will carry passengers within it. Thus it needs to have a large thickness-to-chord ratio and still avoid a strong shock wave recompression as the flow accelerates over it. In addition, the normal Mach number should be as high as possible to minimize the sweep needed for the wing to be efficient.

Studies by NASA Ames [39] and McDonnell Douglas Aerospace West [40], [41] provide good guidance on how many passengers a realistic OFW might carry, how much it might weigh, and at what speeds and altitudes it might fly for the Mach number range 1.3-1.6. These studies derive from and advance earlier work by Kroo [42], Van der Velden [43], Galloway [44] and Waters [45]. Van der Velden's remarkable thesis, which focuses on the oblique wing as the title suggests, considers both oblique and symmetrical swept wings.

Airfoil design

The use of analytic relations for configuration definition using but a few parameters is essential for our generation, and later optimization, of airfoils, wings and even complete configurations. Such a tool has been developed over the years at the DLR by Sobieczky [46]. We use these tools throughout our studies.

We begin with a simplified preliminary airfoil design. A 17.4% thick airfoil section, A, is generated for a flow of $M_\infty = 0.707$ with $c_l = 0.6$. Ideal and a fictitious gas analyses of this airfoil are those depicted in Figure. 3. The data on the sonic line is then used, with the engineering method noted earlier [34] to provide a modified airfoil design. The result is a slightly flattened section with a reduced thickness of 17% shown as airfoil B. The analysis of the two airfoils, A and B, using the MSES program is shown in Figure 4 [47]. Note the variation of the inviscid drag (wave drag) has a minimum at the design Mach number

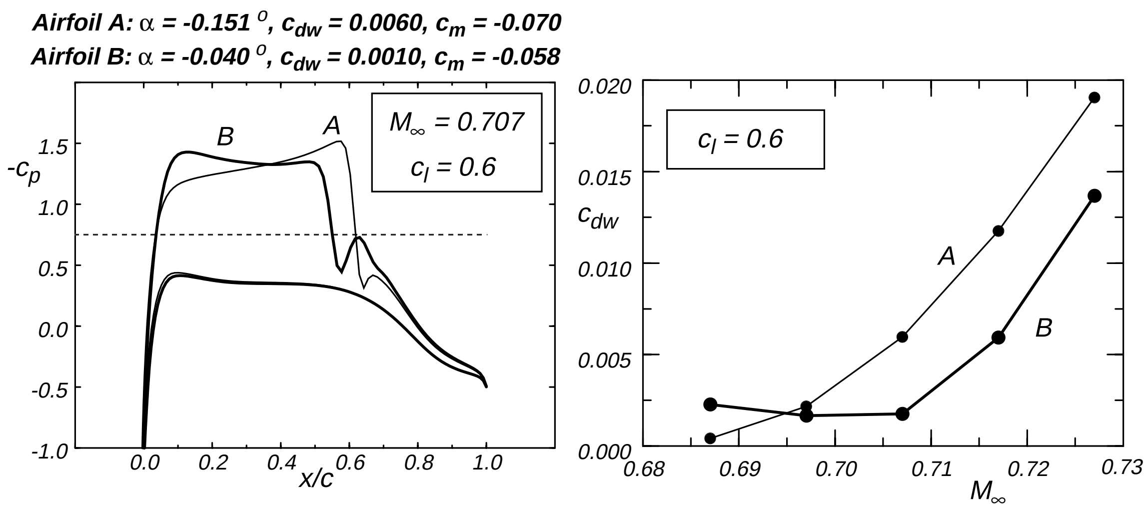

Figure 4. Comparison of an initial airfoil, A, and the redesigned counterpart B.

Inviscid pressure distributions and drag rise for the two airfoils at the design conditions.

and lift coefficient. Airfoil B's wave drag is 50 counts below that of airfoil A. This is indicative of the weaker shock on airfoil B and hence of a reduced retardation of the boundary layer and higher lift for a given drag.

Wing design

The design of a swept, thick, elliptically loaded wing for a supersonic flow is far beyond the scope of this preliminary study. Such a wing, however, provides a good example for the exploration of supersonic design using elliptic or elliptic-like equations. Consequently, we proceed without concern for planform shape, moment coefficients, etc. A flexible set of functions and control parameters allows for the easy modification of the shape to achieve desired ends.

A wing swept to 60 degrees in a $M_\infty = 1.414$ ($\sqrt{2}$) freestream would experience a normal Mach number of 0.707 ($\sqrt{2}/2$). Higher Mach numbers are possible, but this then requires higher sweep angles to achieve the design Mach number. Piloted flight and control become difficult beyond 60 degrees sweep [48]-[50].

For the airfoil considered above we had selected a design lift coefficient appropriate for an OFW transport. The OFW's lift coefficient for its supersonic Mach number will be lower by the square of the ratio of the normal to freestream Mach numbers, or in this case by a factor of 4. Thus we seek a wing with a design lift coefficient, C_L, of 0.15.

We first, however, consider the unswept wing in the normal flow. Our analysis of a resulting wing, without further redesign, is shown in Figure 5. The upper left inset depicts the c_p at various span stations for C_L = 0.6. The upper right inset shows the isotachs at the center and the y ≅ 3 stations. A weak shock is present over about two thirds of the span, indicating that without further redesign this is already a good wing confirming our airfoil B results.

We next analyzed the flow about this untwisted wing swept to 60 degrees in a $M_\infty = 1.414$ flow without modification. This gave a load distribution that was far from elliptical. The lift was largest on the trailing tip where there was also a strong cross-flow shock. While the methodology presented here could now be used to remove this shock wave, this exercise seemed less important than improving the load distribution. Consequently the twist was varied linearly from -6 to +6 degrees from the trailing to the leading tip, with the results shown in Figure 6. We note here the lower pressure coefficients on the trailing wing, indicating the expected higher speeds there. This of course means that symmetrical planform and airfoil section distributions will not be optimum, even if twist is used to obtain an elliptic loading.

These calculations were carried out using a CO-grid, which is not optimal for supersonic flow. Figure 7 shows the isotachs in the wing plane illustrating the bow wave surrounding the leading tip. The isotachs make it clear that a more appropriate grid for these supersonic studies would have an O cross-section like that depicted (see [46]).

5. Conclusions

The method developed here for the design of supersonic

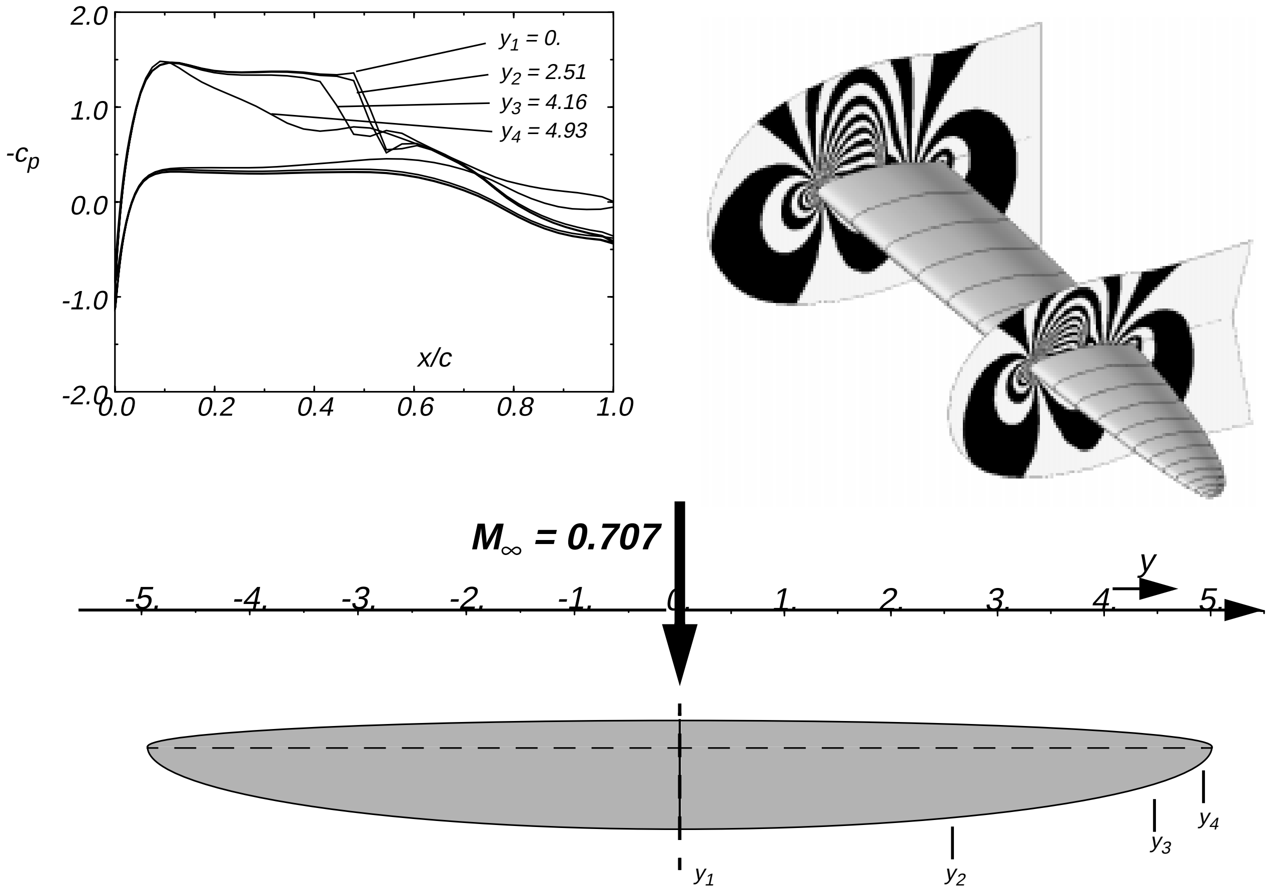

Figure 5. Pressure distributions at four span stations and the isotachs on two grid surfaces for an unswept elliptic wing in an inviscid transonic flow with M_∞ = 0.707, C_L = 0.6:

wings with subsonic leading edges derives from earlier work on supercritical airfoils, wings, and supersonic conical wings. It is applied to a wing flying supersonically swept to 60 degrees in a freestream with M_∞ = 1.414 with a C_L of 0.15. Preliminary results are quite encouraging. For the 17% thick airfoil used in the wing studied here we have been able to delay the drag rise Mach number beyond M_∞ = 0.707, some 15% higher than that previously reported. The results also indicate that an elliptically loaded OFW will be obtained more easily than thought. Lower pressure coefficients on the trailing wing, indicating the expected higher velocities there, mean that even with twist the optimum wing will not be symmetrical about its center plane.

These results set the stage for the application of the method developed here to the full wing in supersonic flow in order to increase its percentage thickness, or normal Mach number, or both. An increase in the percentage thickness allows a reduced chord and span for the same aspect ratio, and results thereby in a smaller wing for the same payload.

Future work will apply this fictitious gas method to the design of conventional symmetric supersonic wings with subsonic leading edges. We stress the importance of advanced information tools, such as the geometry generator used here, to the success of this method as well as to the success of optimization schemes. Because of the ease of, and rational basis for, the geometry generator employed here, optimization studies like those of Cosentino and Holst [15], as well as through expert systems [51] should then follow this redesign of the OFW.

Acknowledgment:

This work was funded by the German Alexander-Von-Humboldt Foundation through a 1991 Max Planck Research Award and by grant from W. Edwards Deming to the third author. Many of the calculations were performed using NASA's Numerical Aerodynamic Simulator facilities. The analysis and interpretation of the results were carried out on DLR workstations. We thank our NASA, DLR, McDonnell Douglas and Stanford colleagues for many fruitful discussions.

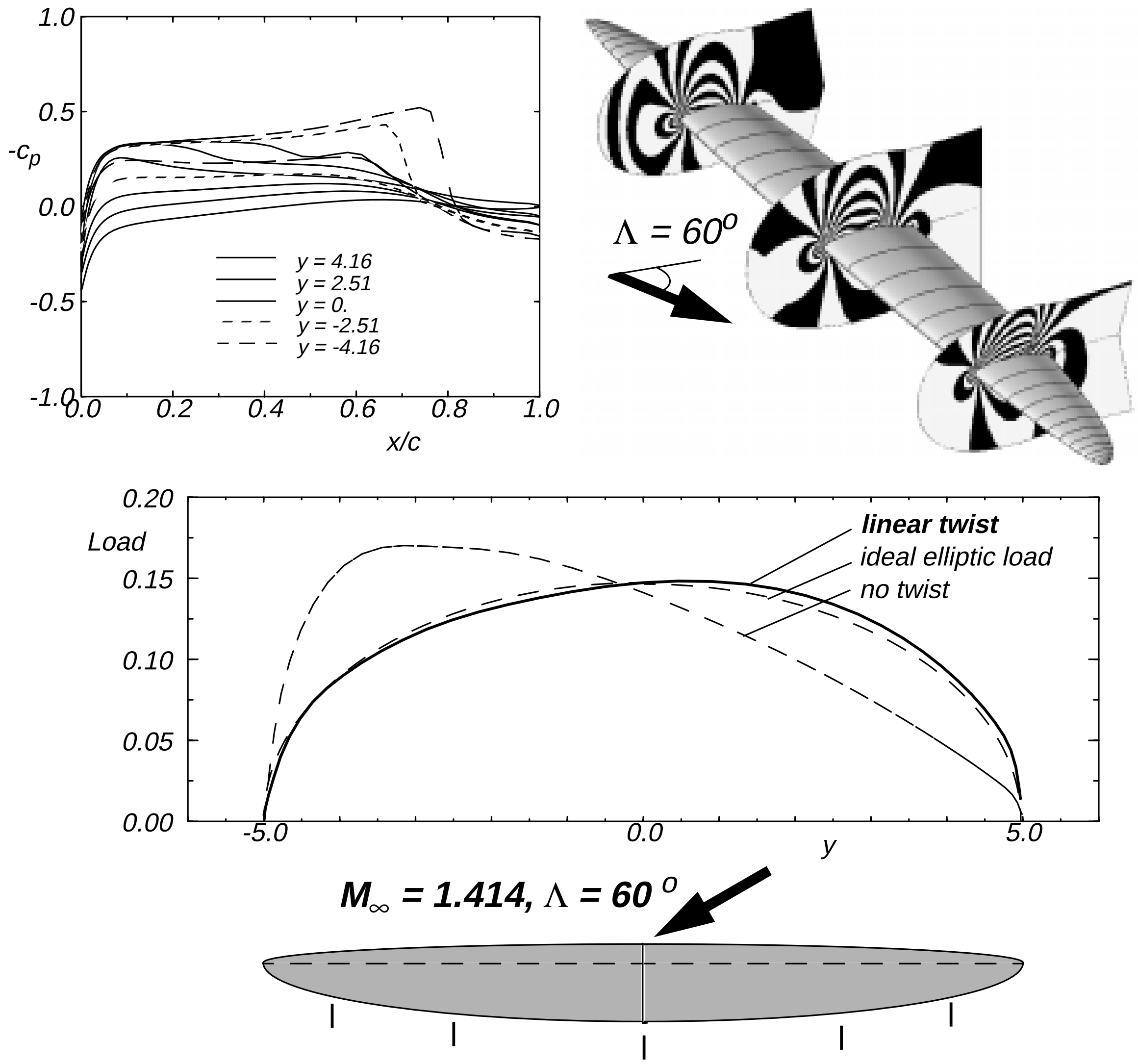

Figure 6. Pressure distributions at five span stations and isotachs on three grid surfaces for a wing swept to 60 degrees in a M_∞ = 1.414 flow with C_L = 0.15. The twist varies linearly from -6 to +6 degrees from the trailing to the leading tip. The load distribution obtained with no twist is also shown.

6. References

[1] Farhat, C., Lesoinne, M., Stern, P. and Lantéri, S., “High Performance Solution of Three-Dimensional Nonlinear Aeroelastic Problems Via Parallel Partitioned Algorithms: Methodology and Preliminary Results,” *J. Computing Systems in Engineering* (in press).

[2] Sobieczky, H. “Research on Inverse Design and Optimization in Germany,” *Appl. Mech. Rev.*, Vol. 41, No. 6, June 1988, pp. 239-246.

[3] Sobieczky, H., “Progress in Inverse Design and Optimization in Aerodynamics, *Conf. on Comput. Methods for Aerodynamic Design (Inverse) and Optimization,* AGARD CP 463, 1989, pp. 1.1-1.10.

[4] Jones, R.T., “Theoretical Determination of the Minimum Drag of Airfoils at Supersonic Speeds,” *J. Aero. Sci.*, Vol. 19, No. 12, 1953, pp. 813-822.

[5] Kogan, M.N., *“On Bodies of Minimum Drag in a Supersonic Gas Flow,” Prikl. Mat. Mekh*., Vol. XXI, No. 2. 1957, pp 207-212.

[6] Jones, R. T., “The Minimum Drag of Thin Wings at Supersonic Speed According to Kogan’s Theory,” *Theoret. and Comput. Fluid Dynamics*, Vol. 1, 1989, pp. 97-103.

[7] Pearcey, H.H., “The Aerodynamic Design of Section Shapes for Swept Wings,” *Adv. Aero. Sci.*, Vol. 3, 1962, pp. 277-322.

[8] Whitcomb, R.T., and Clark, L., “An Airfoil Shape for Efficient Flight at Supercritical Mach Numbers,” NASA TM X-1109, 1965.

[9] Nieuwland, G.Y., “Transonic Potential Flow around

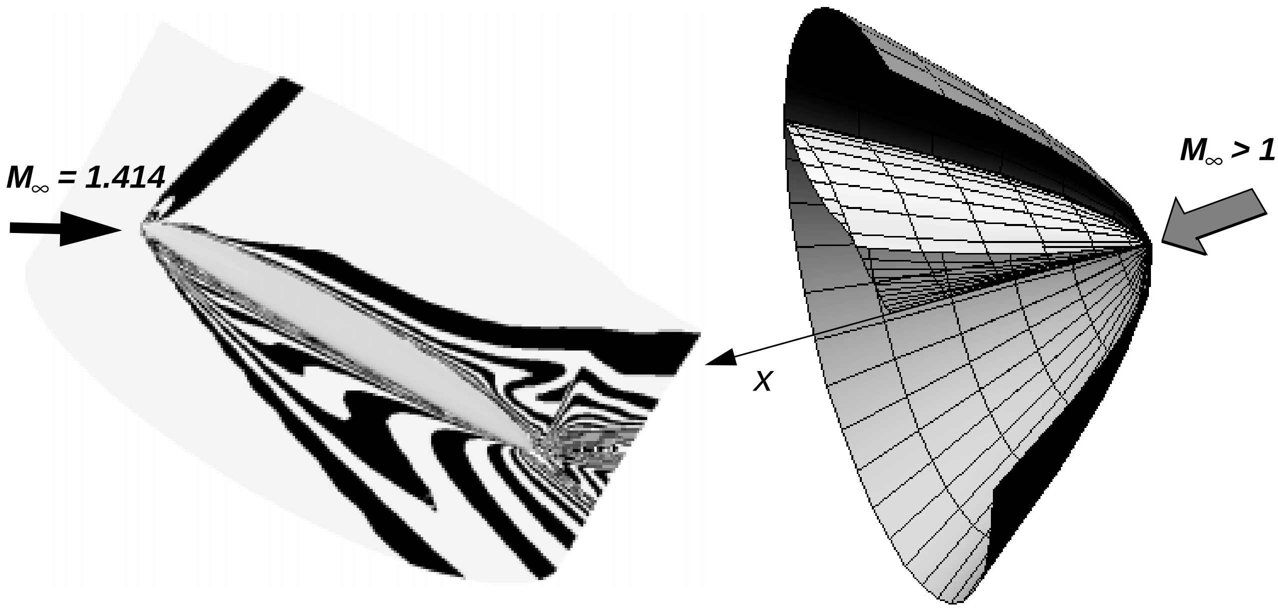

Figure 7. Isotachs in the plane of an elliptic wing illustrating the bow shock wave. $M_\infty = 1.414$ and $C_L = 0.15$. An alternative grid topology is suggested for more refined analysis and design.

a Family of Quasi-Elliptic Sections," NLR TR T.172, 1973.

[10] Garabedian, P.R., and Korn, D.G., "Numerical Design of Transonic Airfoils," *Numerical Solution of Partial Differential Equations*, B. Hubbard, ed., Vol. 2, New York: Academic Press, 1971, pp. 253-271.

[11] Sobieczky, H. and Seebass, A.R. "Supercritical Airfoil and Wing Design," *An. Rev. Fluid Mech.*, Vol. 16, 1984, pp. 337-363.

[12] Sobieczky H., "Die Berechnung räumlicher Überschallfelder," *Z. Angew. Math. Mech.*, Vol. 58T, 1978, pp 331-333.

[13] Fung, K.-Y, Sobieczky, H. and Seebass, A.R., "Shock-Free Wing Designs," *AIAA J.*, Vol. 18, No. 10, 1980, pp 1153-1158.

[14] Sritharan, S.S., "Delta Wings with Shock-Free Cross Flow," *Quart. Appl. Math*., Vol. 43, No. 3, 1985, pp. 275-286.

[15] Cosentino, G.B., and Holst, T.L., "Numerical Optimization Design of Advanced Transonic Wing Configurations," *J. Aircraft*, Vol. 23, No.3, 1986, pp. 192-199.

[16] Niebeck, H.E., Seebass, A.R., and Sobieczky, H., "Inviscid-Viscous Interactions in the Nearly Direct Design of Shock-Free Supercritical Airfoils," *Conf. Comput. Viscous-Inviscid Interactions,* AGARD CP 291, 1980, pp. 3.1-3.10.

[17] Yu, N.J. "Efficient Transonic Shock-Free Wing Redesign Procedure Using a Fictitious Gas Method, *AIAA J.*, Vol. 18, No. 2, 1980, pp. 143-148.

[18] Bock, K-W., "Aerodynamische Optimierung," Dornier Rep. BF 3/87B, 1987.

[19] Raj P., Miranda, L.R. and Seebass, A.R., "A Cost-Effective Method for Supercritical Wing Design," *J. Aircraft,* Vol. 19, No. 4, 1982, pp. 283-289.

[20] Busemann, A., "Aerodynamischer Auftrieb bei Überschallgeschwindigkeit," *Proc. Volta Congr.,* 1935, pp. 328-360.

[21] Busemann, A., "Pfeilflügel bei Hochgeschwindigkeit," Lilientahl-Gesellschaft, Bericht 164, 1943.

[22] Jones, R.T., Wing Plan Forms for High-Speed Flight," NACA TN 863, 1947.

[23] Jones, R. T. "Effects of Sweepback on Boundary Layer Separation, NACA Rep. 884, 1947.

[24] Orlebar, C., *The Concorde Story: Ten Years in Service,* Temple Press, Middlesex, 1986.

[25] Küchemann, D., *A Non-Linear Lifting-Surface Theory for Wings of Small Aspect Ratio with Edge Separation*, RAE Report Aero 2540, 1955.

[26] Maskell, E.C., *Flow Separation in Three Dimensions*, RAE Report Aero 2565, 1955.

[27] Tracy, Richard R., *High Efficiency Supersonic Aircraft,* U.S. Patent No. 5,322,242, June 21, 1994.

[28] Lee, G.H., "Slewed-Wing Supersonics," T*he Aeroplane*, March 3, 1961, pp. 240-241.

[29] Burnet, C., *Three Centuries to Concorde,* Mechanical Engineering Limited, London, 1979.

[30] Pittman, J.L., Bonhaus, D.L., Siclari, M.J., and Dollyhigh, S.M., "Euler Analysis of a High-Speed Civil Transport Concept at Mach 3," *J. Aircraft*, Vol. 28,

No. 4, 1991, pp. 239-245.

[31] Sritharan, S.S., *Nonlinear Aerodynamics of Conical Delta Wings*, Ph.D. Thesis, Applied Mathematics, University of Arizona, 1982. See also: Sritharan, S.S., and Seebass, A.R., “Finite Area Method for Nonlinear Supersonic Conical Flows,” *AIAA J.*, Vol. 22, No. 2, 1984, pp. 226-233.

[32] Fung, K-Y., Sobieczky, H., and Seebass, R. “Numerical Aspects of the Design of Shock-Free Wings,” AIAA Paper No. 79-1557, 1979.

[33] Seebass, R. and Fung, K-Y. “Shock-Free Configurations in Two- and Three-Dimensional Transonic Flow,” *Transonic, Shock, and Multidimensional Flows: Advances in Scientific Computing*, R.H. Meyer, ed., Springer Verlag, 1982, pp. 17-36.

[34] Zhu, Z. and Sobieczky, H., “An Engineering Approach for Nearly Shock-Free Wing Design, *Chinese J. Aeronautics*, 1989, pp. 81-86.

[35] Li, P. and Sobieczky, H., “Computation of Fictitious Gas Flow with the Euler Equations,” *Acta Mechanica*, Vol. 4, 1994, pp. 251-257.

[36] Anderson, W.K., and Thomas, J.L., “Multigrid Acceleration of the Flux Split Euler Equations,” AIAA Paper No. 86-0274, 1986**.**

[37] Thomas, J.L., Taylor, S.L., and Anderson, W.K., “Navier-Stokes Computations of Vortical Flows over Low Aspect Ratio Wing,” AIAA Paper No. 87-0207, 1987.

[38] Jones, R.T. “The Flying Wing Supersonic Transport,” *Aero. J.*, March 1991, pp. 103-106.

[39] Cheung, S., “Viscous CFD Analysis and Optimization of an Oblique All-Wing Transport,” NASA CDCR-20005, 1994. (Private communication to R. Seebass.)

[40] Rawdon, B.K., Scott, P.W., Liebeck, R.H., Page, M.A., Bird, R.S. and Wechsler, J., “Oblique All-Wing SST Concept,” McDonnell Douglas Contractor Report, NAS1-19345, 1994. (Also private communication from B. Rawdon.)

[41] Agrawal, S., Liebeck, R.H., Page, M.A. and Rodriguez, D.L., “Oblique All - Wing Configuration: Aerodynamics, Stability and Control,” McDonnell Douglas Corporation Final Report, NAS 1-19345, 1993.

[42] Kroo, I., “The Aerodynamic Design of Oblique Wing Aircraft,” AIAA Paper 86-2624, 1986.

[43] Van der Velden, A., “Aerodynamic Design and Synthesis of the Oblique Flying Wing Supersonic Transport,” Ph.D. Dissertation, SUDDAR 621, Stanford University, May 1992.

[44] Galloway, T., Gelhausen, P., Moore, M. and Waters, M., “Oblique Wing Supersonic Transport,” AIAA Paper 92-4230, 1992.

[45] Waters, M.H., Ardema, M.D., Roberts, C. and Kroo, I., “Structural and Aerodynamic Considerations for an Oblique All-Wing Aircraft,” AIAA Paper 92-4220, 1992.

[46] Sobieczky, H., Choudhry, S.I., and Eggers, Th., “Parameterized Supersonic Transport Configurations,” Proceedings, 7th European Aerospace Conference, Toulouse, October 1994 (in press).

[47] Drela, M. and Giles, M.B., “ISES: A Two-Dimensional Viscous Aerodynamic Design and Analysis Code,” AIAA Paper No. 87-0421, 1987.

[48] Campbell, J.P. and Drake, H.M., “Investigation of Stability and Control Characteristics of an Airplane Model with a Skewed Wing in the Langley Free Flight Tunnel.” NACA TN 1208, May 1947.

[49] Curry, R.E. and Sim, A.G., “Unique Flight Characteristics of the AD-1 Oblique-Wing Research Airplane,” *J. Aircraft*, Vol. 20, No. 6, 1983, pp. 564-568.

[50] Kempel, R., McNeill, W., Gilyard, G. and Mane, T., “A Piloted Evaluation of an Oblique Wing Research Aircraft Motion Simulation with Decoupling Control Law,” NASA TP 2874, Nov. 1988.

[51] Zores, R., “Transonic Airfoil Design with Expert Systems,” AIAA Paper No. 95-1818, 1995.

[52]