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High Speed Flow Design Using Osculating Axisymmetric Flows

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Abstract:
The introduction of an inverse method of characteristics to the osculating cones (OC) design concept for supersonic flow components extends the options to generate aerospace configurations with supersonic leading edges and inlet diffusors. The concept of osculating axisymmetric (OA) flows yields rotational flow domains from input curved shocks, this way leading to more practical waverider shapes with higher volumetric efficiency.

Keywords:
Supersonic design, Waveriders, Inlets, Inverse design, Rotational flow, Method of characteristics

1. Introduction

Configuration aerodynamic design concepts are aimed at the definition of finding airframe and propulsion component geometries with a controlled flow quality which results in practical parameters like the ratio lift/drag or aerodynamic degree of efficiency. Losses for given lift of an airplane or pressure ratio in a turbomachinery at high speeds are, not solely, but strongly determined by the occurrence of shock waves. In supersonic flow, for inlets and on aircraft, these shocks cannot be avoided but they may be optimized in important applications. Since gasdynamic model equations for compressible flow have become a solid ground for systematic flow modelling, there is a catalog of exact solutions to these equations, either for the mathematical description of experimentally observed local and global flow phenomena, or for the definition of flow sectors with solid boundaries which may resemble practical boundary conditions of aerospace vehicles or their propulsion units. These models for a long time have served for the development of inlet configurations with shock systems and more recently as simple test cases for the development of numerical simulation (CFD) in inviscid fluid mechanics. Such model cases are usually restricted to 2-dimensional plane or axisymmetric flow. An extension to 3-dimensionally bounded flow elements arises from the idea to select arbitrarily shaped stream surfaces in 2D flows and use them as solid boundaries: the flow being still of 2D nature nevertheless is compatible with a 3D boundary condition.

The most wellknown use of this idea is Nonweiler’s use of 2D flows with oblique shocks to define configuration forebodies known as caret wings [1] or, because of their lift-generating mechanism, waveriders. The use of other 2D supersonic flow models allowed an extension of this principle, namely the Taylor-Macoll solution to supersonic flow past a circular cone served to obtain more practically oriented shapes, socalled cone-derived waveriders. Results for such configurations became of revived interest within the past decade when optimization strategies were made possible by the use of fast computers and including viscous effects allowed for realistic prediction of aerodynamic performance. Bowcutt [2] and the school at University of Maryland presented a number of applications of the expanded concept, with increasing emphasis to complex configuration integration of the designed waverider forebody.

Parallel to this development the first author had derived an inverse design concept, originally for 2D transonic flows, which offered a straightforward application to 3D waverider flows: the geometric definition of an oblique shock wave surface in freestream flow and the subsequent determination of the flow field domnstream of this, including the solid surface bounding it. A practical computational design method for supersonic forebody configurations derived from this idea is termed Osculating Cones (OC) method. In the present contribution this concept is briefly explained because the fourth author’s approach to solve the plane or axisymmetric Euler equations in an inverse mode yields a further extension of OC-based configuration shapes, as will be illustrated in the following.

2. Osculating Cones (OC)

Plane 2D as well as axisymmetric flows allow for the selection of special stream surfaces which do not have pressure gradients in the direction normal to them. Hence these surfaces are planes. In 2D flow they represent the

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family of identical and parallel “flow planes”, in axisymmetric flow these surfaces are the family of meridional surfaces including the flow axis.

An idea for finding new flow solutions is to reduce the three-dimensionality of a flow problem to a bunch of selected 2D cases suitably stacked together so that neighboring pressure values are sufficiently close to allow for the approximation of neglecting the resulting pressure gradient. This is, of course, an exact approach for 3D elements of plane or axisymmetric flow. For more general flow elements the boundary condition of a shock with constant strength, i. e. an oblique 3D shock with constant angle in free stream flow will ensure constant (post-shock) pressure level in crossflow planes. This fact is approximating the need of vanishing pressure gradient in the whole flow element well enough to apply a bunching of 2D solutions behind the oblique shock boundary and this way construct the flow element including pressure field and inviscid streamlines. The approximation is very efficient in high Mach number flow especially because the distance between shock and generating surface is small compared to the flow element extension normal to the generating planes.

The axisymmetric conical solutions are such 2D solutions with constant shock strength, with the limiting case of 2D plane flow if the axial distance approaches infinity. Axial distance is defined by the given shock cross section curvature only; using these cone flow solutions to be bunched together ensures a smooth slope surface as given 3D shock surface, the cone flow fields of each individual meridional plane of the cones osculating to the shock surface composes the 3D flow field between shock and a streamline integrated from an upstream location at the shock. Prescribing the axial projection of the shock wave length in each cone flow determines the size of the resulting 3D quasi-wedge flow element, especially its geometric boundary. The concept will be re-iterated in the next chapter describing its extension. Because of the need to select streamlines only from one cone flow Taylor-Maccoll solution, this method is extremely fast and therefore invites to be coupled with optimization strategies [3] and the resulting software is applied to practical aerospace vehicle design examples by various developers.

The OC concept originally was a byproduct of a numerical approach to solve the 3D Euler equations in exact inverse mode for given arbitrary 3D shock waves [4]. Because of its simplicity so far, the less general OC concept has found wider acceptance, a review of these and related activities is given in [5].

3. Inverse method of characteristics

Supersonic flow phenomena are driven by initial and boundary conditions effective within regions of influence and dependence. Classical methods to compute flow elements in the high speed regime therefore are based on the method of characteristics (MOC), baseline approach to computational methods marching in the direction of wave propagation and observing initial and boundary conditions within these regions. A formal application of the MOC, however, allows also for a crossmarching direction, this way computing an inverse problem, illustrated for 2D flow in Fig. 1.

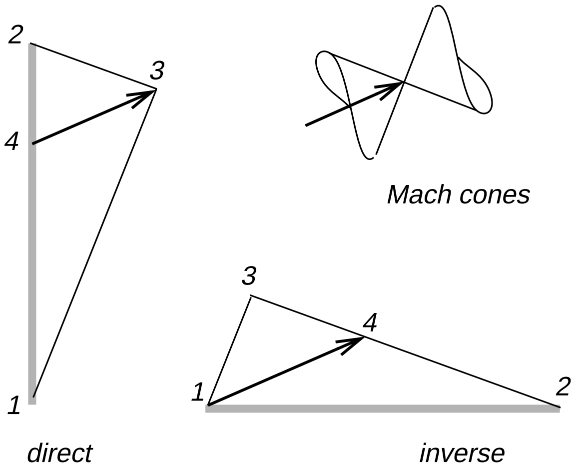

Figure 1. Direct and inverse supersonic flow calculation: Mach cone angle determines characteristic grid.

In the direct approach, initial conditions are given at lines (surfaces) which are inclined toward the streamlines at an angle steeper than the Mach (cone) angle. Given supersonic flow initial conditions at a segment 12 cross section plane are used in this approach to compute the downstream portion 123 of the flow. For rotational flow an iterative entropy update with data 4 is needed.

For flow data given along lines (surfaces) which are inclined toward the streamline at a smaller angle than the Mach angle, characteristic marching is used again for the limited portion of a flow element extending to a direction normal to the streamline. Again, entropy updates are possible by an iterative procedure. This Cauchy problem exists if a flow pattern needs to be imbedded within a known part of the flow, like the post-shock domain behind a given oblique shock wave. The marching is determined by one downstream- and one upstream-directed characteristic. A direct numerical MOC needs to be converted to perform this inverse step instead of the usual direct approach downstream marching. This was carried out [6] with a MOC for rotational flow solving the basic equations, the compatibility relation (1) and the characteristic equation (2) in a meridional plane with coordinates x the flow axis and y the radius:

compatibility relation (1)

$$\pm d\vartheta + \frac{dq}{q}\cot\mu - \frac{\sin\mu\,\sin\vartheta}{\cos(\vartheta\pm\mu)}\,\frac{dx}{y} = \frac{(\sin\mu)^3\,\cot\mu}{\cos(\vartheta\pm\mu)}\,\frac{1}{\gamma-1}\,\frac{dx}{c_p}\,\frac{ds}{dn}$$

characteristic equation (2)

$$dx = \cot(\vartheta\pm\mu)\,dy$$

with q the velocity, θ the flow angle, μ the Mach angle, γ and c_p the ideal gas parameters; ds/dn is the entropy gradient normal to the streamline.

These equations includes irrotational flow if the entropy gradient vanishes. It further reduces to the conical flow o. d. e. if flow parameters are constant along rays y/x = const. and to plane 2D flow for large y, or if the third term in the compatibility relation (1) is omitted. Test cases of the numerical procedure therefore include the wedge solution as well as the Taylor-Maccoll solution: A given straight oblique shock wave in supersonic freestream defines post shock initial conditions for the computation.

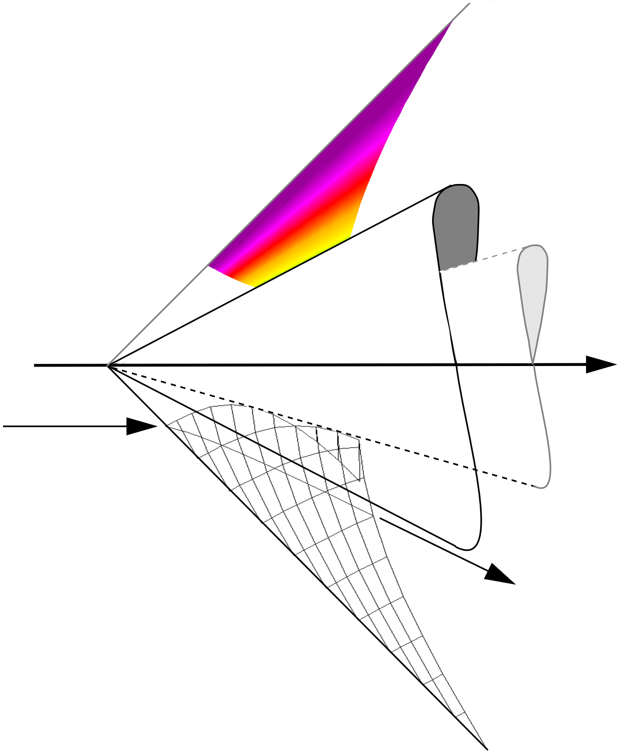

Figure 2. Computing the cone flow field from a given segment of the bow shock. Verifying conical solution structure (pressure isofringes) and occurrence of a limit cone within solid boundary. $M_\infty = 2$, shock angle = 45^o

Fig. 2 shows the cone flow result with an analytical continuation within the solid cone flow boundary: A conical limit surface marks the location of a change of the marching direction and the flow field is folded back onto itself. Such a solution of course is physically not possible, but its occurrence in the inverse MOC alerts the user of the computer code that the given boundary conditions are not compatible with a physical solution within the mathematical domain of dependence. A limitation of the computed region by selecting a streamline as the solid boundary excludes such limit surfaces. Visualization of the flow field shows the conical flow property well represented.

The next step is the modification of the straight line initial condition to an oblique shock with curvature and hence to modified initial conditions with an entropy variation. The example illustrated in Fig. 3 shows that for only slight curvature a substantial change in the local pressure distribution from previously observed conical properties is observed. This makes the parameters for shock geometry definition becoming important design parameters allowing for an effective change of resulting flow elements. Experience still needs to be gained to control the range of parameter variation for surface shape design before unphysical results with limit surfaces appear.

With this inverse MOC developed to be operational, the catalog of axisymmetric flows available for an exploitation to design 3D flow elements may be increased remarkably. In the next chapter we therefore will return to the OC design concept and extend its applicability to waverider or inlet shapes with 3D rotational flow domains.

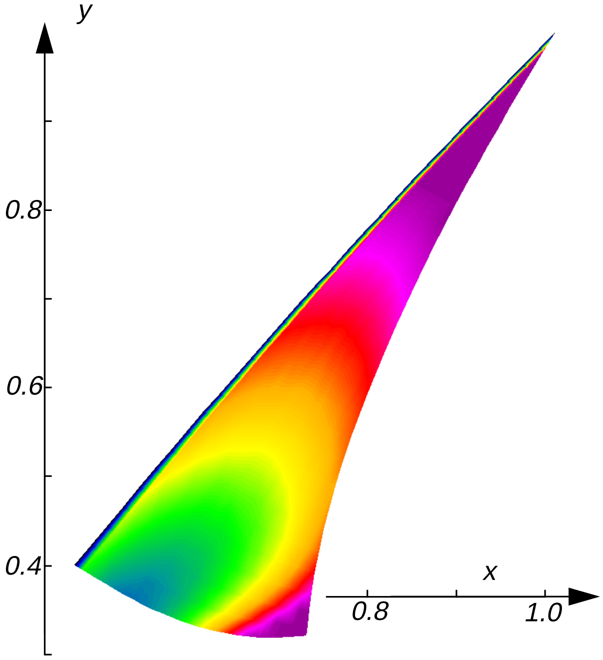

Figure 3. Computing an axisymmetric flow element from given curved shock segment; pressure isofringes. $M_\infty = 2$, shock angle varies between 45 and 50^o

4. Osculating Axisymmetry (OA)

The OC concept to calculate 3D flow fields from using one single conical flow solution may be extended by only a few parameters to arrive at a configuration variation which seems quite promising for practical applications. Fig. 3 illustrates the OC concept with the only extension that the oblique shock (A_0L_{0c}) given in the center plane may have curvature (A_0L_0). With this new function to be prescribed, the whole planform, leading edge coordinates and upper cylindrical free stream surface are defined, along with the shock surface confined between the leading edges. Fig. 3 shows various views of the geometry details: the exit plane view contains the two basic inputs known from the OC appraoach:

(I) Inlet Capture Curve (ICC), with points A_i along its arc length as independent variable to be counted from A_0 in the center plane of symmetry,

(II) Flow Capture Curve (FCC), defined by a function for the distance A_iB_i, drawn normal to the tangent to ICC in A_i.

With ICC described by analytic expressions the geometric locus of the curvature centers (evolute x_i) is defined also. We exclude ICC shapes with inflection points to avoid discontinuities in the evolute. This curve depicts the locus of the individual axes for local axisymmetric flow sections to be designed in each osculating plane ω. An arbitrary section ω_1 is depicted, besides the center plane section ω_0 and the corresponding points in the vanishing plane ω_2 at the configuration 'wing tip'.

For OC flows, shock angle β is needed as one more input parameter (along with the free stream Mach number). Since the input distance A_iB_i is the axial projection of the shock generatrix, the coordinates of the configuration leading edges L_i are defined in 3D space (x, y, z), along with the shock surface of constant slope angle against the free stream direction.

For the new extension introduced here, the shock shape is defined more generally, but by only one curve in the center plane:

(III) Shock Generatrix Curve (SGC), depicted as curve σ in Fig. 3. The slope dz/dx now defines a local shock angle $\beta(x)$, which is maintained for each section as illustrated for ω_1: curve A_1L_1 is a part of the basic SGC A_0L_0, cut off by the FCC projection input.

This latter use of one single SGC to define the whole 3D shock input surface ensures that the postshock pressure in each cross section x = const is the same and thus we can expect the same degree of approximation as it has been found so useful when applying conical flow in each section plane.

With various positions of the local section flow axis, however, we have to compute the Osculating Axisymmetric (OA) flow solution in each section, while the OC concept requires only one cone flow integration and the selection and spanwise distribution of conical flow streamlines. With these being self-similar, only a single basic conical flow streamline is needed and a suitable scaling defines the resulting OC-designed configuration surface.

For OA-designed surfaces the basic OC waverider code has been extended and a few first examples have been computed.

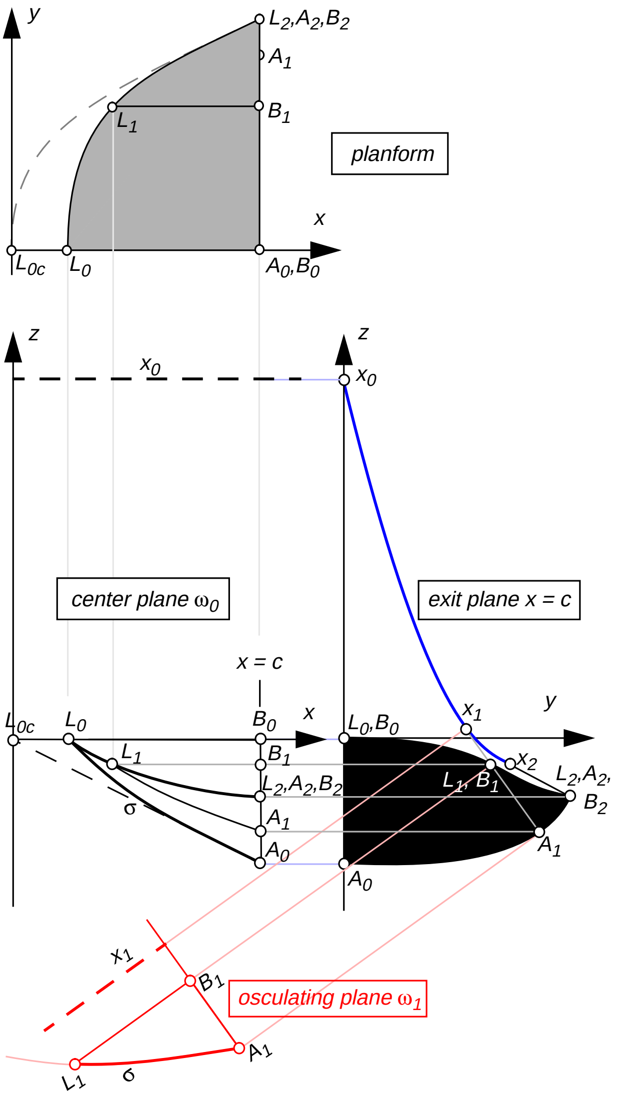

Figure 3. Design concept of Osculating Axisymmetry. Dark domain in exit plane illustrates area of flow capture tube in upstream conditions M_∞

5. Case study

Application of the proven OC procedure with the new extension to an "OA" design concept has been carried out for a few cases of waverider and inlet geometries but still needs a systematic investigation to fully recognize and utilize the options given in addition to the OC variety of generic aerospace configurations.

Based on the test case (Fig. 2) for a cone in supersonic flow $M_\infty = 2$ and a given shock angle of 45^o, a series of examples has been defined for creating inlet shapes with elements of plane 2D flow, axisymmetric flow and a 3D blending between the former, carried out first with the OC concept and subsequently, with the input change of the curved shock geometry, with the new OA approach.

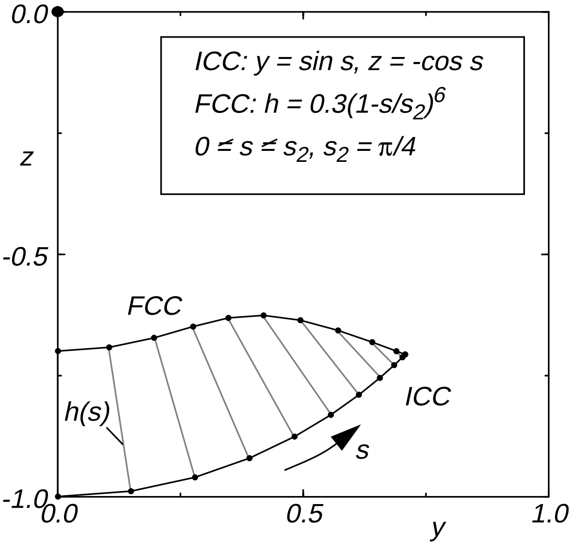

Figure 4. Case study input data for the ICC and FCC curves depicted in the exit plane x = 1.0

Fig. 4 shows the shapes for ICC and FCC which have been chosen for applying the inverse MOC in a finite number of meridional planes for varying portions of an axisymmetric bow wave defined by both a straight and a curved SGC which is illustrated in Fig. 5. A quintic curve (sixth order polynomial) is chosen for the SGC, this way allowing strong control over shock curvature at segment ends. Two isegments L_0A_0, A_0E_0 are chosen for this example connected at the exit plane x station, a certain downstream extension is needed for the shock definition if the flow field is required for all x < 1, especially the surface streamline in the center plane.

Fig. 5 shows an evaluation of the MOC, all integrated streamlines are drawn, including the ones within the configuration surface. This axisymmetric example allows for a selection of the other physical streamlines to be distributed along span, rotated into the meridional plane and originating at the FCC. A cut off at the exit plane completes the surface definition of this 3D supersonic ramp diffusor.

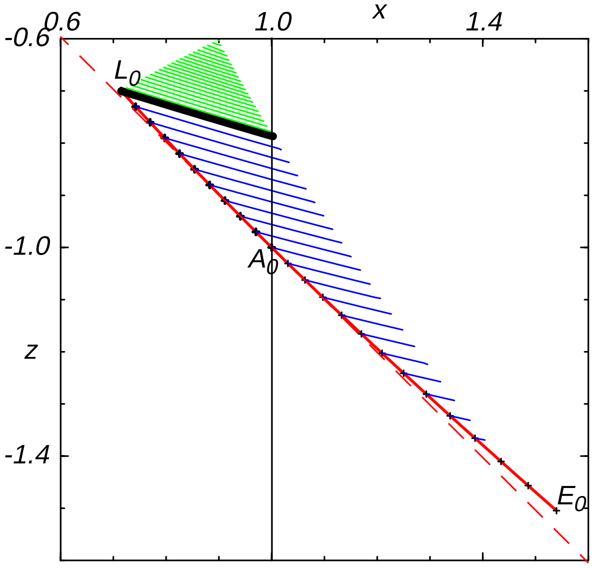

Figure 5. Case study input data for the SGC curve. MOC result for the center plane y=0. Analytical continuation beyond exit plane x = 1.0 to obtain complete flow domain for x < 1.0. also streamlines within solid surface line shown. $M_\infty = 2$

Fig. 6 illustrates the result as a wide, circular arc sector inlet shape. There are only small differences in the resulting ramp surface between conical and curved-shock input but remarkable differences in the flow field quality (Fig.2.)

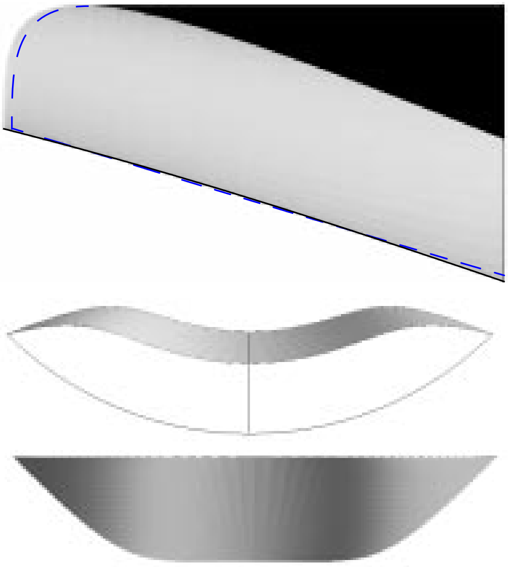

Figure 6. Three-view of the $M_\infty = 2$ inlet ramp shape resulting from input Fig. 4, with constant strength shock. Shape differences for curved shock input Fig. 5, dashed line side view, above. Front view with inlet lip (ICC) , middle, and planform, below. $M_\infty = 2$,

6. Software development

The previously developed OC design concept has been implemented in various computer codes to design waverider forebodies and inlet shapes. The required input functions for ICC and FCC are generated by the toolbox of a geometry preprocessor [7] which provides data for CFD grid generation: Design results are input for commercial software unstructured grid generation which is required also for a new DLR Euler and Navier Stokes flow analysis code. Alternatively, resulting surface geometry data are input for standard CAD systems to design and manufacture wind tunnel models for experimntal investigations. Finally, a powerful postprocessing of the results with graphic visualization is added for rapid qualitative analysis of the results.

An interactive version of the OC design code is currently expanded to include also the OA concept, so that the new results may be processed as proven for the OC work. Parameter variations will result in the immediate visualization of the corresponding shapes illustrating the success of proper optimization strategies or simulating a process of mechanical adaptation. Fig. 7 illustrates 3 OC-designed inlet shapes, which are generated from a set of parameters with only a few of them needed to smoothly blend one result into the other, bridging the gap between plane 2D and axisymmetric flow.

Based on the fast computation, the goal is to develop software for rapid pre-design tools in the form of an aerodynamic expert system for high speed flow components.

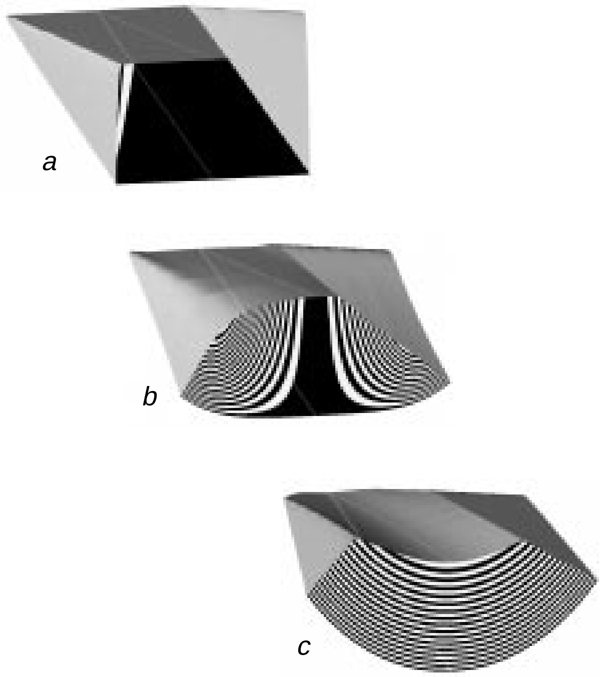

Figure 7. Inlet shapes with equal flow capture area, pressure isofringes in exit plane, $M_\infty = 2$.

7. Conclusion

En extension to an inverse design method for supersonic flow components has been presented which allows for the prescribing of variable strength shock waves in a quasi-2D approach with the method of characteristics. Rotational flow fields with controlled entropy distribution this way may be used to ectend the parameter range for waverider and inlet geometries, first results with the new method show design variations with reduced length and higher volumetric efficiency. Future activities include expert system software development for high speed aerodynamic design.

8. References

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