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On Conical Flow

H. Sobieczky
DLR German Aerospace Center

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H. Sobieczky
DLR German Aerospace Research Center
D-37073 Göttingen, Bunsenstr. 10
Germany

Helmut.Sobieczky@dlr.de

Abstract

Transonic conical flow is modelled by near sonic (potential) flow theory to study singular solutions to the Taylor-Maccoll (T-M) equation which have been used for explaining high speed flow phenomena.

A numerical Euler method of characteristics (MOC) in inverse mode is shown to not only modelling T-M results but also flows with prescribed, curved shock waves.

Recent 3D configuration design methodology based on the Osculating Cones (OC) technique is extended to an Osculating Axisymmetry (OA) concept using the inverse MOC thus allowing for waverider design with refined flow control.

Introduction

Design methodology of 3D high speed aerospace vehicle components recently makes extended use of inviscid models for plane and axisymmetric, transonic and supersonic flow elements with shocks. In a time when fast desktop computers can evaluate 2D inviscid compressible flows within seconds, these solutions become of renewed value for initial steps to shape realistic 3D flow boundaries where the flow parameters come as a result with the geometric shape. Busemann [1] gave exact solutions to plane and axisymmetric compressible flow which have guided the aerospace design community in understanding the parameters needed to arrive at configurations with improved aerodynamic performance. Among basic flow models, axisymmetric conical flows play an important role in providing known flow models for configuration design. Taylor and Maccoll [2] gave ordinary differential ("T-M") equations for conical flow which have been widely used to model flows emplying conical parts. Moelder [3] pointed out some special solutions to these basic equations which might be useful for the design of internal flow components. In these flow models, however, the occurrence of singularities gives rise to doubts about the physical relevance of certain solutions to the T-M equation.

Such singularities are observed also in transonic flows: Exact solutions to the basic equations for transonic flow have been found mainly in various hodograph formulations. Guderley [4] shows the mathematical background of transonic similarity solutions but only certain ones are applicable to model flows of practical relevance. Much later, the present author has tried to gain insight to transonic flow phenomena using hodograph formulations when the advent of large computers already suggested numerical solution methods to these strongly nonlinear problems. An attempt of presenting the 'knowledge base' resulting from such studies for the development of transonic and high speed design methodology is made in [5].

To the author's knowledge, conical flow in the transonic domain has not been modelled by the classical (small perturbation theory) similarity solutions, supposedly because of the T-M equation already providing solutions in the full range of ideal gas flow Mach numbers. In a first chapter of this paper, free parameters in the transonic similarity solutions are identified to include also those describing conical flow. Some new solutions will be shown in comparison to well-known ones like the sonic throat. Singularities like those occurring in T-M solutions are found.

In the following chapter conical (supersonic) flow is modelled by Y. J. Qian's method of characteristics in an inverse mode which provides solutions to flows with prescribed shocks. These may also be curved and this way lead to rotational flow patterns with more general properties than conical shocks of sonstant strength may yield. The numerical algorithm is therefore an inverse Euler code for axisymmetric flow design with shock control.

Finally, such generalization of conical and other flow models is proposed to serve for an update of the author's concept to design supersonic waverider configurations: In the past decade the method of Osculating Cones (OC) has been adopted widely to develop efficient computer codes for numerical design and optimization of waveriders. This progress is reviewed in [6]. Replacing the T-M equation in this software by the inverse method of characteristics allows for a fairly arbitrary definition of high speed aerocomponents like hypersonic aerospace vehicles, high

* Sr. research scientist, Sr. member AIAA

speed aircraft with supersonic leading edges and the shaping of supersonic inlets. Fast methods and case studies with parameter variation should help developing dedicated geometry preprocessors using functions tailored from the present models.

Conical flow within the near sonic knowledge base

Theoretical models for 2D plane inviscid transonic potential flows have most successfully been developed 30 - 40 years ago using the hodograph transformation which removes the strong non-linearity of the basic equations of fluid motion. Most of the problems of the mixed elliptic - hyperbolic type model equations can be studied and understood by finding particular solutions to the linear hodograph equations. Sobieczky prefers a special hodograph transformation ("Rheograph"). For flows with Mach numbers close to unity and small perturbations this approach leads to coupled potentials for both geometrical variables and the variables of state and includes also axisymmetric transonic flows [5]. This system is illustrated in the following, a harmonic solution family is recalled and shown to include also conical flows, which may not have been observed previously.

Near sonic equations for plane and axisymmetric flow

In plane 2D or axisymmetric flow the (meridional) plane is identified by coordinates (x, y), the flow velocity components are (u, v) (Fig. 1).

$$\begin{aligned} x &= l \cdot X \\ y &= l \cdot \sigma^{-p_2 \cdot (1+2p_1)} \cdot (\gamma+1)^{-3 \cdot p_2/2} \cdot Y \\ \frac{u}{u_{ref}} - 1 &= \pm\left(\sigma^{(1+p_1-(1-p_1)\cdot p_2)} \cdot |U|^{(1-p_2)}\right) \\ y^{p_1} \cdot \frac{v}{u_{ref}} &= l^{p_1} \cdot \sigma^{(1+p_1)} \cdot (\gamma+1)^{(3 \cdot p_2 \cdot (1-p_1)/2)} \cdot (1-p_2) \cdot V \end{aligned} \quad (1)$$

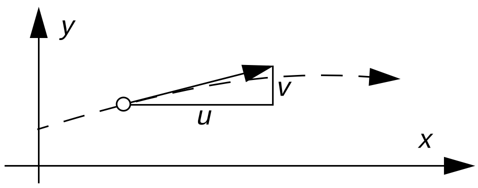

Fig. 1. Similarity transformation of physical variables (x, y, u, v) to (X, Y, U, V) with scaling data l and σ. Switch parameters p_1, p_2 control plane /axisymmetric and linear / near sonic flow. Ideal gas parameter is γ, u_{ref} for near sonic flows is the critical sound speed.

A similarity transformation allows to use reduced variables X, Y and U, V where any combination of geometric and gasdynamic scaling values l and σ can be used to define suitable similarity parameters.

$$\begin{aligned} V_t - Y^{p_1}(s,t) \cdot U_s &= 0 \\ V_s \mp Y^{p_1}(s,t) \cdot U_t &= 0 \end{aligned} \quad (2)$$

$$\begin{aligned} X_s - U^{p_2}(s,t) \cdot Y_t &= 0 \\ X_t \mp U^{p_2}(s,t) \cdot Y_s &= 0 \end{aligned} \quad (3)$$

Gasdynamic equation and compatibility relation result in two systems of Beltrami equations (2), (3) in a parameter (rheograph) plane (s, t). The whole system of four first order P.D.E's is linear for p_1 and/or p_2 equal to zero but nonlinearly coupled in the axisymmetric transonic case where $p_1 = 1$ and $p_2 = 1/3$. The system is elliptic for U < 0 and hyperbolic for U > 0. With independent variables s,t a sonic line U(s, t) = U* = 0 (in Fig. 2, at s = 0) may be chosen to fix the type change, similarly the location of the axis $Y(s,t) = Y_o = 0$ may be prescribed.

Combining solutions of the elliptic and hyperbolic system along the sonic line U = s = 0, so that in domain s < 0 typical elliptic boundary value problems and in s > 0 typical hyperbolic initial value problems are formulated. Data compatibility is needed along a contact line AB on the t-axis (Fig. 2).

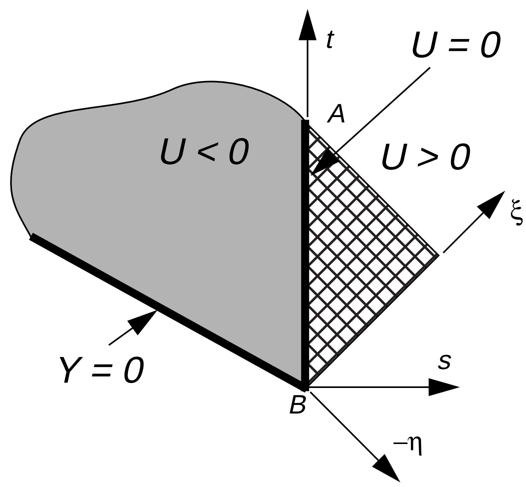

Fig. 2. Combined solution for the elliptic (subsonic) and hyperbolic (supersonic) equations, with common data (X, Y*, V*) along the sonic locus U* = 0: Initial values define solution within triangular domain of dependence H. Characteristic variables ξ, η.*

For U > 0, system (2), (3) can be transformed to characteristic variables

$$\xi = t + s \qquad \eta = t - s \qquad (4)$$

and results in system (5), (6):

$$V_\xi - Y^{p_1} \cdot U_\xi = 0$$
$$V_\eta + Y^{p_1} \cdot U_\eta = 0 \qquad (5)$$

$$X_\xi - U^{p_2} \cdot Y_\xi = 0$$
$$X_\eta + U^{p_2} \cdot Y_\xi = 0 \qquad (6)$$

These equations can be used for a numerical integration of the supersonic problem defined by initial values along the sonic line as has been done for transonic airfoil design [5].

Quasi-harmonic potential flow models

Guderley's work suggests to find a family of particular solutions to the model equations by a quasi-harmonic ansatz in polar coordinates for the 4 coupled variables::

$$U = r^n \cdot h(\varphi)$$
$$V = r^{n + p_1 \cdot b} \cdot k(\varphi) \qquad (7)$$
$$X = r^{b + p_2 \cdot n} \cdot f(\varphi)$$
$$Y = r^b \cdot g(\varphi)$$

with solution parameters *n, b.* A system of coupled ordinary differential equations for the functions *h, k, f, g* results

$$f' = C_1 \cdot g \cdot |h|^{p_2} + D_1 \cdot f$$
$$g' = C_2 \cdot f \cdot |h|^{-p_2} + D_2 \cdot g$$
$$h' = C_3 \cdot k \cdot |g|^{-p_1} + D_3 \cdot h \qquad (8)$$
$$k' = C_4 \cdot h \cdot |g|^{p_1} + D_4 \cdot k$$

with coefficients C_i and D_i listed in (9) and (10):

h < 0 (subsonic flow): (9)

$$C_1 = -b$$
$$C_2 = b + p_2 \cdot n$$
$$C_3 = n + p_1 \cdot b$$
$$C_4 = -n$$

$$D_1 = 0$$
$$D_2 = 0$$
$$D_3 = 0$$
$$D_4 = 0$$

h > 0 (supersonic flow): (10)

$$E = cos(2(\varphi - \varphi^*))$$

$$C_1 = b/E$$
$$C_2 = (b + p_2\ n)/E$$
$$C_3 = (n + p_1\ b)/E$$
$$C_4 = n/E$$

$$D_1 = -(b + p_2\ n)/E$$
$$D_2 = -b/E$$
$$D_3 = -n/E$$
$$D_4 = -(n + p_1\ b)/E$$

Near sonic similarity solutions

It can be shown that only the ratio

$$\mu = n/b \qquad (11)$$

is relevant for varying the structure of the resulting flow model. For subsonic plane linear flow (p_1, p_2 = 0) the obtained solutions are equivalent to results of conformal mapping

$$U - i \cdot V = (X + i \cdot Y)^\mu \qquad (12)$$

Before identifying an applicability of this family of solutions to conical flow modelling, the welknown flow model describing the accelerated flow through the sonic throat of an axisymmetric Laval nozzle is illustrated here, resulting from a numerical solution of the coupled O. D. E's (8) with (9) and (10). The solution parameter is $\mu = 3$ and we choose $n = 1$ and $b = 1/3$.

The flow model is one of the few which has a closed form solution for the variables U, V in the plane X, Y:

$$\pm U^{\frac{2}{3}} = 2aX + a^2Y^2$$
$$V = 3a^2XY^2 + \frac{3}{4}a^2Y^4 \qquad (13)$$

Integrating the system for h, k, f, g with a 4th order Runge Kutta method requires care at the location of 3 singularities in system (8):

1. Starting the integration at $\phi = 0$, with $g = 0$ representing the axis Y = 0, requires setting up compatible initial conditions for (8), with $g = 0$, $g' > 0$, $h < 0$, $h' = 0$.

2. Arriving at the sonic locus, where $h(\phi^*) \to 0$, will lead to gradient steepening of $g(\phi)$. Step size reduction easily overcomes this difficulty. The value ϕ^* where the sonic locus will be reached, depends solely on choice of the exponents n, b.

3. Arriving at the limiting characteristic $\phi_{lim} = \phi^* + \pi/4$ will, in general, lead to singular (logarithmic) behavior because of the denominator $\cos(2\phi)$ in the coefficients (10). This singularity mainly determines if the chosen parameter μ yields a flow model of practical value. As seen in Figs. 3 and 4, this solution is regular at ϕ_{lim}.

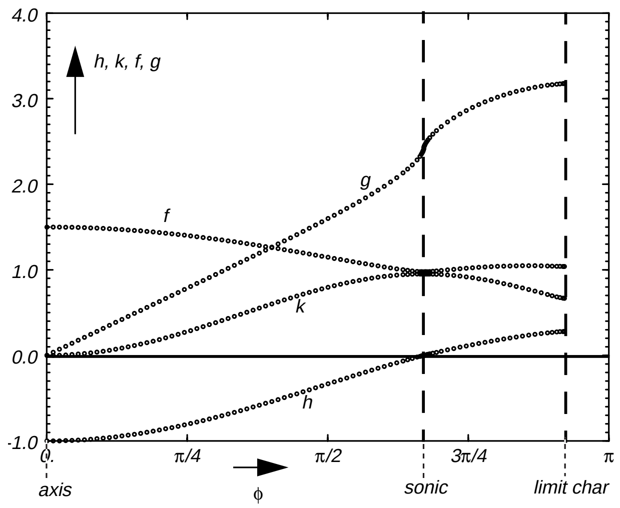

Fig. 3. Functions h, k, f, g for the sonic throat example within the domain of subsonic flow and supersonic flow upstream of the first limiting characteristic.

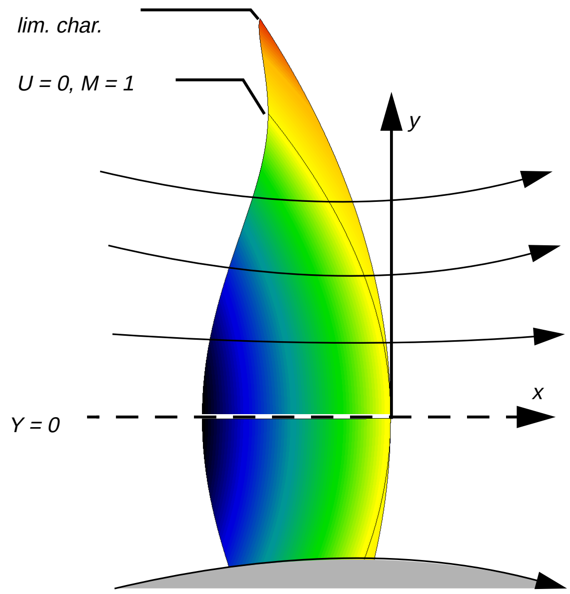

Fig. 4. Visualization of the solution for the sonic throat model. The flow is regular at the limiting characteristic and continues smoothly further downstream.

Near sonic conical flow

Topological analysis of the axisymmetric, near sonic solutions (7) using a free parameter μ, results in a choice of

$$\mu = 0 \qquad (14)$$

which is obtained here with n = 0 and b = 1, to model the flow types of our current interest, namely those with a conical structure. Some examples with various initial conditions have been computed to verify known cone flows.

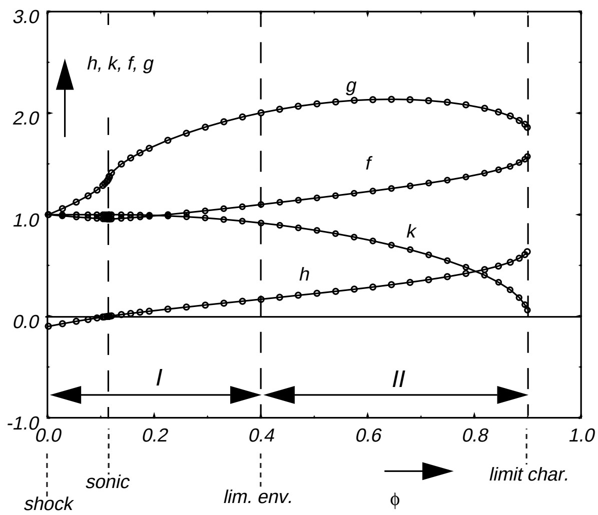

Fig. 5. Functions h, k, f, g for a conical flow example. The solution contains two flow models: (I) transonic accelerated and (II) supersonic decelerated conical flow

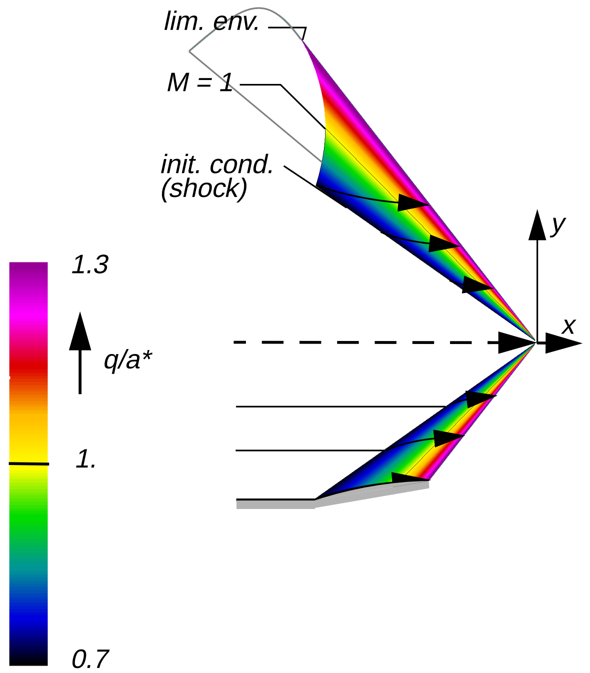

Fig. 6. Conical accelerated flow: Flow domain bounded by subsonic inlet conditions (for instance past conical shock) and singular exit condition with infinite gradients.

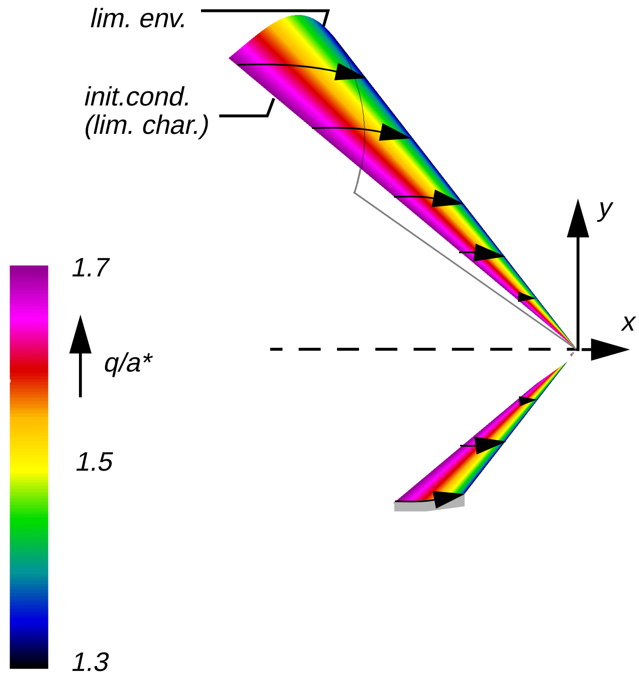

Fig. 7. Conical decelerated flow: Flow domain bounded by nearly parallel supersonic flow inlet conditions and singular exit condition with infinite gradients.

The example illustrated in Figures 5 - 7 reveals two parts of the solution which are connected along an exit cone surface but cover the same (upstream) part of the flow space. This exit cone surface occurs as an envelope of characteristics, in Fig 5 at values of ϕ well ahead of the location of the limit characteristic. Both parts of the flow terminate at the envelope cone, with no solution existing downstream. Therefore we conclude that such a solution does not model physically possible flows.

Mölder et al. [8] use a result to the Taylor - Maccoll equation equivalent to the first part of the solution to study the structure of shock reflection at the axis: The result of their work as well as from the present transonic modelling is that no straight oblique shock can approach the flow axis. In theory a sink line swallows the flow shortly downstream at the limiting envelope, in numerical modelling with a refined CFD analysis as well as in experiment a Mach disk accompanies the shock reflection.

The second part of the solution is related to Busemann's shock diffusor [1]. Supposedly a tuning of the initial conditions for integration of system (8) shows that also this physically possible solution is included in the present modelling tool.

Axisymmetric flows with conical and curved shocks

In the past chapter the construction of transonic conical flows was performed using a mapping procedure derived from the 2D hodograph method. Initial conditions in this method define shocks, sonic surfaces and limiting characteristics. This is an inverse approach, defining flow properties and compute the geometry. In the following, the method of characteristics developed by Qian is used in inverse mode to compute similar flow fields but these are solution to the compressible flow Euler equations in the whole supersonic Mach number range. The inverse approach is related to compute the flow within the characteristic triangle of Fig. 2: computation is not carried out downstream but in a cross - flow marching procedure [6].

The inverse approach yields the computation of folded solutions similar to those obtained from the transonic model above. A first result obtained was the computation of cone flow, including the analytical continuation with a limiting cone hidden within the cone body. Presently, we investigate solutions like Mölder's [3] internal conical flows: Computation beyond the enveloping cone is no problem with this approach, a result is the flow field for both parts of the solution as found with the transonic model, see the characteristics domain depicted in Fig. 8.

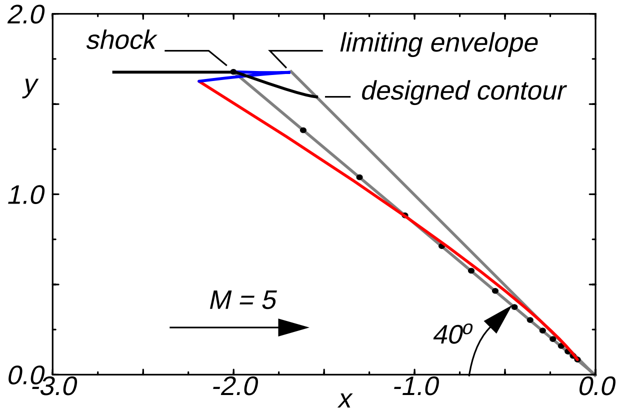

Fig. 8. Internal conical flow model with oblique shock approaching the axis, computation of the flow past given shock, resulting in contour streamline, limiting envelope and fold-back solution.

Curved shock waves

A major extension of solutions obtained beyond conical flows with the inverse method of characteristics is the option to prescribe curved shocks as initial conditions for the downstream flow computation [6]. Flows with controlled entropy distribution may be constructed this way.

An example for a flow with a curved shock wave and the resulting flow field streamlines is illustrated in Fig. 9: The inclination of the convex input segment of the shock wave varies between 40 and 50 degrees, in an unperturbed flow of M = 2. Among all streamlines within the characteristic triangle of dependence the resulting contour shape is included, which generates such shock wave.

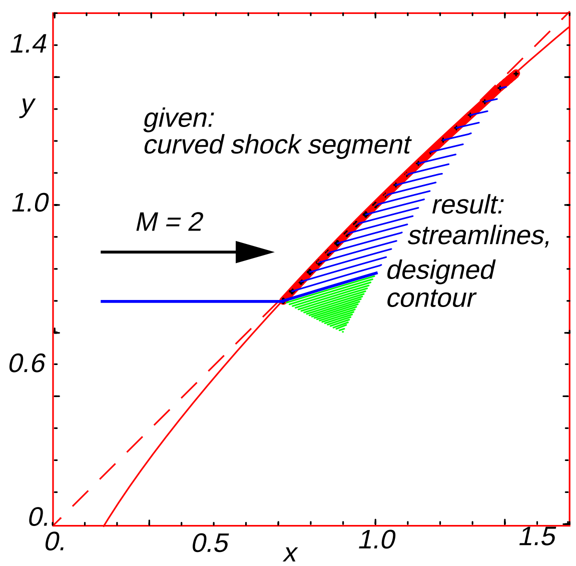

Fig. 9. Computation of an axisymmetric flow with curved shock wave. Shock angle varies from 50^o to 40^o

Concepts for 3D configuration optimization

With updated experience in interpretation of numerical solutions to both the Taylor-Maccoll equation and the inverse method of characteristics, this knowledge base in designing arbitrary axisymmetric compressible flows with prescribed shocks may be used successfully for flexible supersonic configuration design based on extensions to the classical waverider concept.

Waverider design concepts

Waveriders are configurations obtained from exploiting known analytical or numerical solutions to the equations of inviscid flow motion, by suitable selection of stream surfaces and shock waves as physical boundaries. Examples for plane and axisymmetric flows serve as known solutions, with more recent solutions to inverse problem formulations greatly enhancing the variability of the obtained configurations. Some techniques to arrive at quite realistic configurations for aerospace vehicles and supersonic inlets are given in [6], namely the concept of Osculating Cones (OC) which seems to have found acceptance in various aerospace projects in their early design phase.

Fig. 10 shows some typical 3D inlet shapes which can be designed with the OC method: From known simple 2D wedge to axisymmetric (cone) flow all intermediate forms of 3D flow components may be created using the OC approach. The idea is based on the fact that constant obliquity of the 3D shock surface (geometrically a “slope surface”) defines post shock deviation in the osculating plane with its constant azimutal angle along the streamwise coordinate.

Observing the local axisymmetry of the resulting 3D flow by engaging conical flow within the osculating plane is equivalent of a higher order approximation to the correct 3D inviscid flow equations, which is usually rewarded by excellent CFD verification using a reliable CFD Euler code. Viscous flow corrections by boundary layer methods are used to arrive at realistic lift/drag ratios which are verified reasonably by Navier-Stokes analysis. For applications of the OC method see ref. [6].

Osculating Axisymmetry

Developments of rapid and robust design of waveriders beyond the reported applications of the OC method are based now on our inverse design method for flow components with curved shocks. The ideas of OC remain valid if the shock strength remains constant in cross sections but not necessarily along the freestream flow axis. An oblique shock chosen as meridional curve (“shock generatrix curve” SGC) and rotated observing curvature of the given “inlet capture curve” (ICC) in the inlet cross sectional

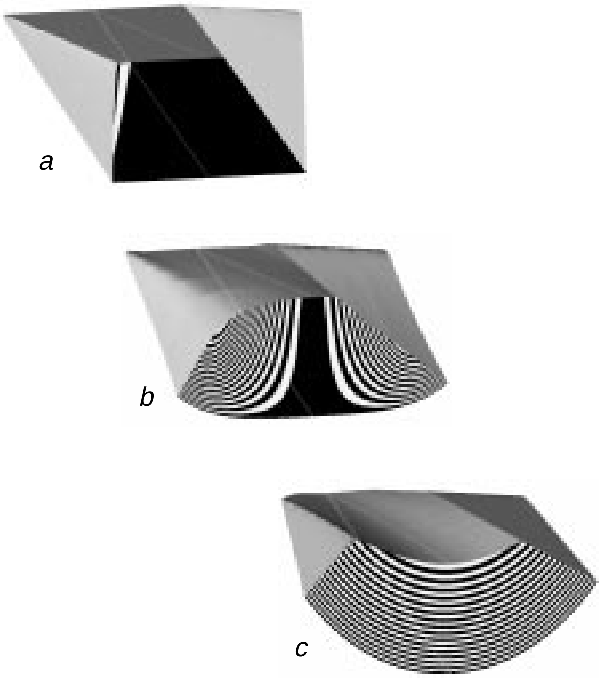

Fig. 10. Examples of inlet design using the Osculating Cones (OC) concept: Plane 2D wedge flow (a), conical flow (c) and an OC blending between a and c resulting in a geometry (b) with strong inlet flow quality control (see isobar fringes)

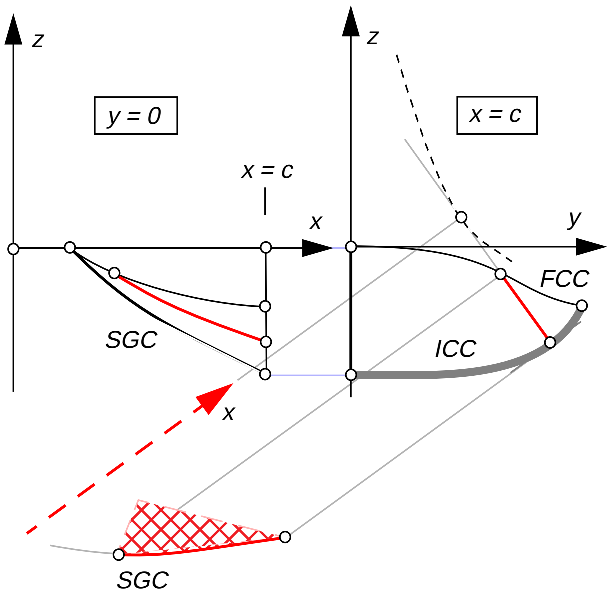

Fig. 11. Design concept of Osculating Axisymmetry. Given shock generatrix curve SGC, inlet capture curve ICC and flow capture curve FCC defines shock wave. Application of inverse method of characteristics in selected osculating planes. Varying axial distance defined by ICC curvature.

plane, defines local axisymmetry and definition of the leading edge projection ("flow capture tube" FCT) results in the used axial length of the SGC at different spanwise sections. See Fig. 11 for an illustration and Ref. 9 for more details of this "Osculating Axisymmetry - OA" approach.

First results of the method confirm the wellknown fact that curved shocks reduce axial length which is useful for reduction of viscous drag. Refined optimization strategies based on OC software may result in better configurations using the OA concept.

Conclusion

The family of near sonic potential flow similarity solutions has been extended by identifying solution parameters for conical transonic flow. Case studies shed light in singular solutions to the Taylor-Maccoll equation which have been used for interpretation of some experimental and numerical results. Such refined flow models can be obtained also be an inverse method of characteristics which is the new tool for extending an operational 3D configuration design concept (Osculating Cones) to model shocks with varying strength. Software with such refined tools allow geometry definition of flexible waveriders, wings with supersonic leading edges and inlets.

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