SENSITIVITY OF AERODYNAMIC OPTIMIZATION TO PARAMETERIZED TARGET FUNCTIONS

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ABSTRACT
Aerodynamic design of high speed airfoils and wings is carried out by a new Genetic Algorithm software and applied to novel configurations like the Oblique Flying Wing. The goal is achieved by establishing a flexible input data generator for both direct and inverse design: geometry and flow quality (pressure distribution) is modelled by a set of analytical functions with parameterized input. The role of these functions and choice of parameters strongly influence the results which is shown here for the Pareto fronts of airfoils and wings with optimum aerodynamic performance. An existing manual design case serves as test case for the new optimization software.

KEYWORDS
Aerodynamic design, Genetic algorithm, Optimization, Pareto front, Airfoils, Wings, New configurations

INTRODUCTION
Accumulated knowledge and experience with aerodynamic design through the past decades offers a chance to arrive at optimum configurations in an accelerated approach: Previous manual design and many practical case studies have contributed to the existence of starting data for new design efforts which now are supported by automatic computerized optimization strategies. A key issue to arrive at useful results seems a suitable parametric definition of previous knowledge, to be available for automatic routines and subsequently applied for the manifold of new case studies.
Several novel aircraft concepts, like wing-only configurations (the Blended Wing Body for transonic and the Oblique Flying Wing for supersonic flight) challenge the designer: While theory points out a substantial potential for improvements (not only in aerodynamic performance), traditional tools need to be adapted to the novel shapes under study.
In this contribution we use a flexible geometric shape definition tool to create various airfoils and wings which allow for geometry adaptation towards optimum aerodynamic performance. Geometry preprocessing for CFD analysis within a Genetic Algorithm optimization method [1]
gives a set of airfoils or wings to be analyzed with numerical flow analysis (CFD) and successively modified and selected, to arrive at a number of ‘fittest’ configurations which we so far use to learn and update our knowledge basis. Subsequently some selected optimum case studies will be used for more sophisticated analysis and design studies.

PARAMETRIC INPUT DEFINITION

To start our aerodynamic optimization study, we need a geometry and scalar function generator for both direct and inverse design input data definition: The goal is to have a maximum flexibility for data generation with a minimum of control parameters. For aerospace applications, airfoils have always been the basic 2D shape element, to compose 3D wings. Mathematical functions generating such components are therefore needed especially when they include suitable adjustment parameters in order to arrive at desirable shapes in a reduced time of needed iterations. In the following we present our own tools for such efforts.

Basic functions and composition to arbitrary curves

Composing arbitrary curves in 2D and their subsequent use in 3D space advises to have a toolbox of model functions available, from which all specific curves and distributions may be generated. We have set up a catalog of basic functions in the unit square with free parameters for slopes and curvatures (or power series leading terms, alternatively, see Fig. 1a). These basic elements (cubic and quintic splines, and other relatively simple functions with special mathematical structure: “Ramp”) are used to compose arbitrary curves with strong shape control, Fig. 1b. They are used also in distribution functions for computational algebraic grids and for control of unsteady shape definition (4D geometries). Various interactive versions to create geometries have been developed by different users of this technique, here we need only the fact, that a set of support points with gradient and curvature control, but also shape influencing between these support points by choice of the basic functions is achieved. See [2] for examples and references on this work.

Figure 1. A set of normalized basic functions (a) to be scaled and used for arbitrary parameterized function definition (b): Strong shape control in the support points.
Airfoils and wings

The most important application of a geometry tool for aerospace is its ability to create such basic shapes like airfoils, wings and cross sections. Analytical airfoil definition is attractive since the days of the NACA airfoil catalog; today we need refined shapes for designing wings with improved efficiency. Our basic shape elements and curve compositions were used to define shapes directly, by specifying practical data like leading edge radius, thickness and camber distribution, and an extendeable number of parameters defining the details at the trailing edge. We call this airfoil family “PARSEC” airfoils [3], more specific “PARSEC-XX” with a number of additional parameters for stepwise refinement of the design:

$$Z_{PARSEC} = \sum_{n=1}^{6} a_n(p) \cdot X^{n-1/2} + \sum_{n=1}^{3} b_n(c) \cdot X^n + \Delta Z_{dte} + \sum_{n=1}^{2} Z_{bump}$$

We have used the basic set with 11 parameters $p$ for 3D wing iterative manual design with the parameters defined by spanwise distribution modeled also by our basic functions, see Li et al [4]. Oyama et al [5] has shown PARSEC-11 to suit optimization strategies very well: This family of airfoils was found to be superior to several other known airfoil generation methods like for instance a generalized Joukowsky mapping method.

In Ref. [1] a different Joukowsky generalization for optimization is proposed which makes use of the curve generation illustrated above. The airfoil is generated as a parameterized function $r(\phi)$ in the mapping plane $\zeta = z + 1/z$, with a number of sections divided by support points with the abovementioned control in slope and higher derivatives, see the sketch Fig. 2. In this contribution we want to point out the sensitivity of aerodynamic performance toward the chosen model function parameters.

![Figure 2](image-url)

Figure 2. Joukowsky mapping for generating aerodynamic airfoils (a). Generalization of Joukowsky circle for arbitrary airfoils: Choice of n sections with individual parameters, (b)
**Inverse design target functions**

While airfoils and wing geometries need to be parameterized suitably to finally arrive at some acceptable performance (which will be simulated by an accurate Navier-Stokes solver), the input of a favorable pressure distribution should implicitly contain knowledge of desirable flow quality; optimization will focus on a best approximation to the input target function. This may be done by an iterative optimization like for varying the geometric shape, or it is achieved by an inverse approach like Takanashi’s method [6].

In a careful approach we have used a result from a CFD analysis to create a model pressure distribution using our parameterized function elements in a routine we call 'PARCPX-20'. Fig. 3 shows both the CFD result [4] and the somewhat ’idealized’ cp distribution for different span stations of the wing.

Other examples are inviting to check both the direct and the inverse approach: Analytical results for slender airfoils plus their pressure distribution in the transonic domain [7] can be duplicated by the PARSEC-14 parameters for the airfoils and by a very simple set of PARCPX parameters for the surface pressure model.

![Figure 3. Pressure distributions on wing sections: results from a CFD simulation (a), remodeled by functions observing main structure of data distribution (b)](image)

**AERODYNAMIC OPTIMIZATION WITH GENETIC ALGORITHM**

At this point we present some results from the first author’s thesis [8], which is based on the generalized Joukowsky parameterization of airfoils with an extension to lifting wings. Investigations to learn about the sensitivity of aerodynamic parameters to the choice of input parameters and chosen basic model functions were a main goal of computing examples. We used inviscid flow simulated by the Euler version of a CFD code for the individual flow analysis runs, a few cases were investigated using the Navier Stokes version of the code.

**Pareto front**

A Genetic Algorithm was chosen as optimization strategy, target function was simply the lift-to-drag ratio and for most of the studies shape optimization in inviscid flow was attempted. Results are best viewed in the drag, lift diagram: the whole population of computed cases fill the space below a Pareto front, which finally contains the best results, Fig. 4:
We note that a Pareto front is similar to an individual case study’s drag polar which suggests the
design point at an optimum lift/drag ratio (point $B_{opt}$ in Fig. 5) which theoretically is not situated
on the Pareto front. With all optima along the Pareto front there is one ‘optimum-optimorum’
(point C) which is in fact the one single optimum point of all design studies for the target function
lift/drag.

![Figure 4. Optimization toward Pareto front of airfoils with improved Lift/Drag ratio](image)

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![Figure 5. Pareto front is the envelope of the whole population’s drag polars](image)

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CASE STUDIES

In the following study we apply various model functions for generating a 'population' of 100 different airfoils using an arbitrary choice of values for the geometry generation parameters defining the mapped airfoil curve \( r(\phi) \). A delimiting routine keeps input data within practical boundaries which will give reasonable shapes. This accelerates the design process to fewer generations necessary for arriving at the pareto front.

Inviscid flow results

We use 4 sections for the \( r(\phi) \) curve (see Fig. 2b), divided equally to length \( \pi/4 \) each. A constraint for resulting thickness eliminates airfoils thinner than 17%. Comparisons with an existing study let us choose a transonic Mach number \( M_\infty = 0.707 \). A first comparison of different model functions illustrates that 'Cubic' and 'Quintic' splines and the 'Ramp' function yield different Pareto fronts (Fig. 6a): The Ramp function seems suited more for higher lift coefficients than the Quintic spline, but they both seem to guarantee better results than the Cubic spline. We are still far from being able to generalize such findings but eventually we will be able to restrict the use to most suitable model functions.

![Figure 6](image.png)

Figure 6. Sensitivity to types of parameterization: Pareto fronts and lift-over-drag ratios for using 4 sections with Cubic splines (a), Quintic splines (b), and Ramp functions (c). \( M_\infty = 0.707 \)

Next we of course want to learn more about the family of optimum airfoils: Fig. 6b tries to illustrate the pressure distributions for some airfoils along one Pareto front. We see that for lift coefficients with nearly negligible wave drag we observe shock-free airfoils while above \( c_l \approx 0.8 \) a recompression shock wave develops.

With the reservation that Euler code results will only give inviscid wave drag which in real viscous flow will interact with the boundary layer so that Euler results have only limited value, we nevertheless perform inviscid off-design computations for a few airfoils. We see in Fig. 7 that for a higher design lift the optimum condition becomes much more a 'point design': optimum conditions take place in a very narrow interval of angle of attack (\( \alpha = 1.46^\circ \) for \( c_l = 0.76 \)). The other depicted drag polar shows a less sensitive design with optimum near \( c_l = 0.51 \).
Viscous flow results

Realistic aerodynamic performance data will be obtained by using a reliable Navier/Stokes code for flow analysis. Compared to inviscid Euler computations this will be much more timeconsuming; in Fig. 8 one example for a Pareto front resulting from Navier/Stokes code analysis is compared to the respective Euler results. Now the selection of an optimum airfoil C with best lift/drag ratio is applicable: Maximum $c_l/c_d$ is reached for $c_l = 0.7$; the shock-free flowfield is depicted.

Figure 7. Off design conditions (Euler analysis) for selected airfoils along Pareto front.

Figure 8. Pareto fronts of Euler and Navier/Stokes CFD results (a). Selection of optimum lift/drag airfoil: Isofringes local Mach number showing shock-free viscous flow
Wings

A straightforward extension to the third dimension of varying airfoils as wing sections along span is the parametric definition of the basic wing geometry data like leading and trailing edge shape, twist and dihedral. Practical requirements usually put restrictions on such basic properties of the configuration, like on the thickness of the airfoils to be designed. In an exercise testing the GA optimization strategy for 3D wings we challenge existing manually optimized results in transonic and supersonic flow, allowing the wing sections to be varied and optimized for various Mach numbers.

A first example is shown for a subsonic Mach number $M_\infty = 0.707$ and an untwisted elliptic wing with $C_L \sim 0.55$. Only a weak spanwise variation for the resulting wing sections is obtained. The result is verifying the classical knowledge for incompressible flow that a single airfoil along an elliptic planform ensures minimum drag by a spanwise elliptic distribution of the circulation. Spanwise shape variations of the wing sections occur mainly on the upper surface and must be attributed to an automatic minimization of shock wave drag on the wing upper surface: Indeed we see that the optimization work found a shock-free pressure distribution along span, see Fig. 9.

![Figure 9. Optimization of an elliptic wing in transonic inviscid flow $M_\infty = 0.707$: shock-free pressure distribution, visualization of the sonic bubble on the upper wing surface.](image)

The next example is aimed at comparing the results of GA optimization with the manually optimized result for an Oblique Flying Wing (OFW) in supersonic flow [4]. This OFW was found by using classical aerodynamic theories for swept wings, minimum drag bodies and supercritical
airfoils. Compared to the symmetrical planform in subsonic unswept flow (Fig. 9), a sweep-back of the trailing tip was found beneficial. This non-symmetrical planform (Fig. 10) was a fixed input for the GA optimization, but the spanwise airfoil shape variation and the twist distribution was allowed to adjust.

The target was again just improving lift/drag ratio, hence we got a Pareto front of wing shapes, see Fig 10, within lift coefficients of 0.07 and 0.23. Most interesting is the comparison with the single manually obtained design with \( C_L = 0.145 \): It sits exactly on the Pareto front which indicates that within the modeling of inviscid flow, the use of the classical aerodynamic knowledge base for the manual design has been sufficient for this application. On the other hand, we have seen that different model functions may influence the Pareto front so that there may still be some potential for improvement of this purely inviscid design case. Also, of course, a variation of the planform within practical constraints should also be done in future investigations.

**Figure 10.** Oblique Flying Wings in swept supersonic inviscid flow \( M_\infty = 1.414 \), sweep angle \( \lambda = 60^\circ \). Pareto front and ratio lift/drag: Comparison with a single manually designed example [4].

**Inverse approaches**

The next step is a GA optimization of a parameterized pressure distribution, starting with an idealized result of CFD analysis for the manual design example as illustrated in Fig. 3. The goal is then minimizing a least square deviation of resulting pressures from the given model function. This approach has the advantage that there is already much more of our knowledge base included implicitly than in just optimizing the ratio lift/drag: Lift coefficient is prescribed this way as in-
integral and drag can most favorably be influenced by the type of pressure distribution. Results of this work will be given in a forthcoming paper.

While this approach may be called ‘inverse’ because of prescribing the desired result by a ‘PARCPX’ model and, through our flexible ‘PARSEC’ shape generator hopefully will be obtained without much compromising, a ‘true inverse’ concept like Takanashi’s method [6] needs PARCPX for definition of the design goal and PARSEC for an initial CFD computation; the design code then provides corrections to the PARSEC input based on differences between the PARCPX input and the CFD result for the pressure distribution. A project to develop also such a method is under progress.

CONCLUSION

With having developed a Genetic Algorithm optimization algorithm for aerodynamic applications we have performed test case studies focussing on providing suitable input data. These are flexible shape functions for airfoil and wing geometries as well as for pressure distributions. Direct optimization of global aerodynamic efficiency (lift/drag) or inverse optimization finding geometries for prescribed detailed local flow properties can be performed with these shape functions. The new technique can suitably be applied to early phase design studies of innovative configurations like wing only configurations.

An example of manual optimization serves as test case to learn more about sensitivities of the proposed geometry and flow quality parameterization using Euler CFD, before more costly Navier-Stokes computations are being used in the optimization efforts.

REFERENCES