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Inverse Euler Solutions for High Speed Aerodynamic Design Problems

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Abstract

This presentation reviews, illustrates and expands an applied method to compute compressible flow patterns in three-dimensional space which are suitable for flight vehicle configuration design.

1 Introduction

Inverse procedures to obtain desirable results in engineering design have originally been defined what today is called the “true inverse” approach: The mathematical background of a problem in engineering mechanics is reformulated, i. e. its system of equations is modified (“inverted”) to allow using as input the desired resulting performance in detail. For aerodynamics this means application of accumulated practical knowledge, that certain pressure distributions in flow fields not only result in target forces (e. g. lift on a wing) but also more detailed in the behavior in off-design operating conditions, where the target function should have some controlled, if necessary, deviation of the optimum design behavior.
While we nowadays have certain successful “true inverse” reformulations in the fluid mechanic basic equations, more complex applications claim for a wider definition of targets in design, frequently becoming necessary because of multidisciplinary considerations of “what is optimum performance in the practical world?”. Inverse tools are therfore more and more replaced by repeated “direct approach” try-and-err computations to arrive at a target function, which is of course more costly but with increased computer power this seems to matter less. Optimization strategies to speed up the process for arriving at a given target, say, pressure distribution on a wing to remain in aerodynamics, are therefore becoming more important tools which seem to blend better into any multidisciplinary work with model equations from, say, structural mechanics, thermomechanics, acoustics, etc.
What is therefore left of the values of true inverse methods, at least in design aerodynamics? Certainly a better understanding of phenomena is developed by the engineer, but also, if used in combination with optimization strategies, an accelerated arrival at desirable results seems likely, because of a problem - oriented definition of free parameters allowed to adjust for optima, and avoiding optimization runs searching in parameter spaces where no solution can be existing.
Mathematical models of compressible flow describe the phenomena and practical consequences occurring in high speed aerodynamics. Main efforts need to be taken to shape flight vehicles and turbomachinery components to avoid strong shock waves because of, first, their causing of wave drag and, second, interacting with viscous effects near the surface decreasing lift forces besides adding further friction drag. This way, the inviscid flow phenomenon of shock waves is causing dramatic decrease of aerodynamic efficiency defined by the ratio of lift-over-drag.
In this contribution a hitherto not used combination of inviscid flow modelling by the Euler equations of fluid motion with “true inverse” reformulation of boundary value problems is given for some transonic and supersonic applications, to help learning about sensitivities of shape parameterization, which subsequently should support computational optimization methods in aerodynamics.

2 Local high speed flow patterns of practical interest

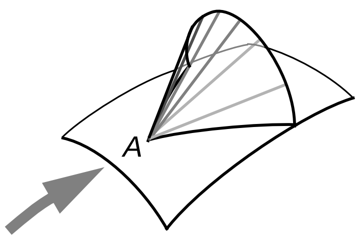

Figure 1: Region of dependence downstream of point A

Unlike flows with locally subsonic Mach numbers, where each location in the flow field is influenced by all other locations and flow parameters there, supersonic flows have bounded regions of influence and dependence: If a given flow example therefore is altered slightly at a certain point A (Fig. 1) on the flow boundary, changes in the flow will be felt only within a domain bounded by a Mach cone. For stronger alterations in A and downstream of it, the Mach cone will become a shock wave, with stronger changes in the flow field but unchanged flow upstream of the shock wave. This is basic knowledge in supersonic fluid mechanics, in the following some non-trivial applications to transonic and supersonic aerodynamics will be made.

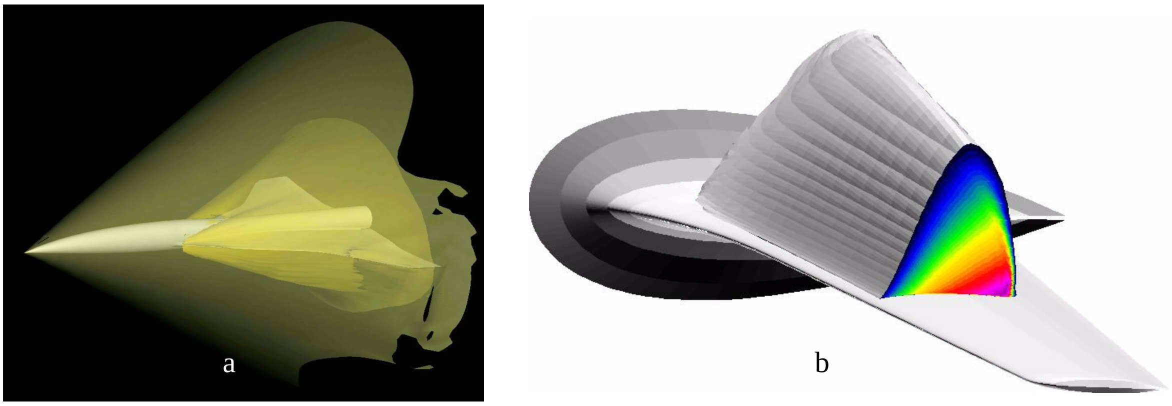

Figure 2: Flow domain visualization for aerodynamic configurations:
CFD results for an SST wing-body configuration in supersonic flow (M = 2.4), with shock waves emanating from body tip and from wing root apex (a), and for a wing in transonic flow (M = 0.82), with supersonic bubble embedded in subsonic flow and cut open to show iso-Mach fringes (b).

2.1 Supersonic Flow

The sketch Fig. 1 occurs in real flows past supersonic aircraft at various locations on the configuration surface: Figure 2a shows visualization of the results of computational simulation with an Euler CFD code. A system of shock waves displayed here for shock strength above a certain threshold, revealing that a bow wave is created by the tip but also another shock wave emanates from the wing root at the body. Challenging design tasks exist for such aircraft: Shock formation at the wing root can be reduced by a filleted integration of wing and body and a careful tailoring of the cross section area of fuselage plus wing. An inverse design approach would be the geometrical description of a shock system with desirable wave drag and from it find the flow field plus the surface geometry compatible with this shock system. For this example, the approach would be mathematically ill-posed but there are supersonic applications shown in the following chapters, which very well allow this inverse approach without running into numerical difficulties stemming from ill-posedness.
Before showing such applications, the more difficult example of transonic flow will be illustrated because it sparked ideas how to treat local flow domains bounded by surfaces with known flow property mathematically.

2.2 Transonic Flow

In transonic flow past a configuration both supersonic flow (M > 1) and subsonic flow (M < 1) occurs, separated by a surface with smooth transition of M from <1 to >1. Also, non-smooth transition from M > 1 back to subsonic velocity occurs if the bounding surface is a shock wave. Fig. 2b shows the visualization of a computed local supersonic domain on a wing in flow with high subsonic Mach number: The flow accelerates smoothly to supersonic velocity but is returning to subsonic velocity passing a recompression shock (steep slope of the visualized domain with M > 1). For a given flight Mach number, occurrence and strength of this shock is solely dependent of the body shape, for flight Mach numbers not too close to unity there is a chance to suppress the shock and return with gradual deceleration to subsonic flow. It is a primary task of design aerodynamics to develop systematical methods of shape generation with shock-free flow quality or at least with shocks of a controlled strength. In the following, the author’s inverse methods, with a more recent application of Euler CFD, will be outlined.

3 Inverse Marching

Transonic flows are the first examples to illustrate the concept of inverse design within a limited domain of the flow field. The task is to reshape the local supersonic flow domain so that it will exhibit a shock-free recompression. A systematic method to achieve this goal for 2D (airfoil) flow is a transformation of the basic model equations into the variables of state u, v (the velocity components in 2D space, also called the Hodograph working plane). The model equations then become linear and the boundary value problem for a typical transonic airfoil flow suggests the new procedure of marching of a given sonic flow conditions locus.

3.1 Choice of the Independent Variables

This procedure is illustrated in Ref. [1] in detail, with several applications to shock-free airfoil flow. The point with shock-free flows is that they are isentropic, hence allow for the simplification of Euler model equations to the compressible flow potential equation: for a long time the most paractical form of applied problems model equations to be solved by then smaller computers. Figure 3 shows the principle of solving the boundary value problem in the hodograph plane: We define an elliptic boundary value problem E in the subsonic part (M < 1) of this plane and solve the linear Poisson equation for this domain E. Practically we do this by first extending the elliptic problem into the supersonic part M > 1 and evaluating the solution along the (sonic locus) AB. We then solve a hyperbolic problem with initial values along AB taken from the elliptic solution and put together the valid elliptic solution in M < 1 with the newly computed hyperbolic solution representing supersonic flow within a local domain. Without going into the details here we note the fact, that a *fictitious* extension of the subsonic problem delivered data for a sonic surface and initial values for a hyperbolic computation, marching away from the sonic locus AB and obtaining a field H with a boundary which connects to the elliptic boundary smoothly but is (slightly) different to the initial continuation. We note that the marching direction for the hyperbolic procedure is not in the streamwise but in the increasing Mach number direction, with ending at a resulting boundary which will be transformed back to the physical plane as a surface compatible with shock-free flow.
In the following we learn from this two-step procedure of steeing up an elliptic boundary value problem first and the adding an inverse marching procedure resulting in a flow boundary. The practically interesting fact, that the resulting flow is shock-free, stems from the elliptic first step procedure, pre-conditioning the whole problem to result in a flow without a recompression shock

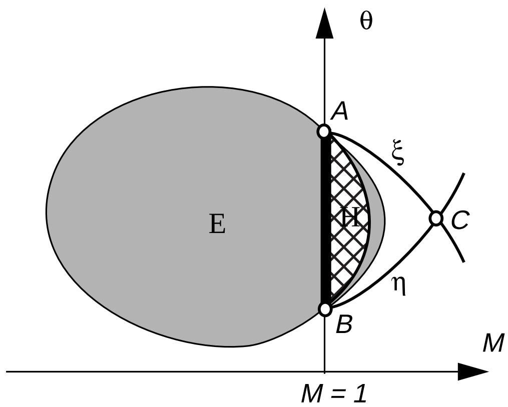

Figure 3: Transonic 2D flow mapped to the hodograph plane: Suggesting inverse marching starting from the sonic locus $\overline{AB}$

3.2 Characteristics: Direct and Inverse Marching

What has been simply termed here “Marching procedure”, in practice is a numerical method of characteristics (MOC) which first was developed for the potential flow simplification of isentropic flows (which are valid for shock-free flows), but later was extended to flows with variable entropy, thus being an Euler accurate method of characteristics [2]. Figure 4 shows the difference between a direct and an inverse step of the MOC: in direct approach, we have data in basically an inflow initial condition $\overline{12}$ and we compute the triangular region 123 dependent of it. With variable entropy along this initial condition, but being constant along the streamlines in resulting flow direction, there needs to be an update from data at point 4, which requires an iterative procedure for this non-linear model equation. It is the basic element of a direct marching starting from some given upstream flow conditions.
Analysis of the marching in the hodograph plane teaches us that here an inverse marching step is performed, with initial data along, or relatively close to, a streamline ($\overline{1'2'}$ in Fig. 4, right hand side) . Marching is performed now sideways in a crossflow direction, resulting in a flow field 123 compatible with the predefined data along $\overline{1'2'}$. Again the Euler accurate approach will update with entropy convection along streamline $\overline{14}$.

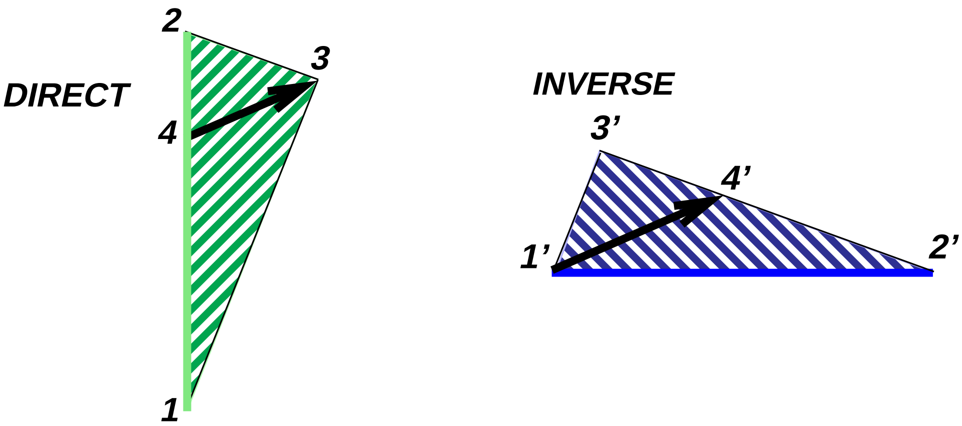

Figure 4: Direct and Inverse basic marching element for method of characteristics

3.3 Inverse Euler Method of Characteristics

An efficient Euler MOC computer code for inverse applications has been written by Y. J. Qian, it has been applied for plane 2D and axisymmetric compressible flows. The extension of the 2D potential equation concept to axisymmetric flow is represented by the third term in the compatibility relation (1), entropy convection is modelled by the right hand side term of (1). The characteristic equation (2) stays the same as for potential flow MOC:

Compatibility relation

$$\pm d\vartheta + \frac{dq}{q}\cot\mu - \frac{\sin\mu\ \sin\vartheta}{\cos(\vartheta\pm\mu)}\frac{dx}{y} = \frac{(\sin\mu)^3\ \cot\mu}{\cos(\vartheta\pm\mu)}\frac{1}{\gamma-1}\frac{dx}{c}\frac{ds}{dn} \qquad (1)$$

Characteristic equation

$$dx = \cot(\vartheta\pm\mu)dy \qquad (2)$$

3.4 Integration of the 3D Euler Equations

Applications of the axisymmetric MOC to obtain practical 3D flows will be outlined later, but basically we still do not have a general purpose fully 3D inverse MOC. Already in the early time of using the potential equation for transonic flows, ther was an effort to generalize the inverse marching concept to starting at arbitrary sonic surfaces in 3D space, to compute 3D local supersonic domains as illustrated in Fig. 2b, with removed or controlled recompression shocks. This was first done for the 3D transonic small perturbation equation [3]. Fig. 5 shows the given initial data for such approach: a surface z(x,y) and flow parameters distributed along it. The goal equivalent to inverse marching using the MOC is computing a flow solution adjacent to this boundary z(x,y) with restrictions accepted concerning the extension of this solution far into the flow field. From the 2D examples and the hodograph plane we have learned that smooth analytical quality of the initial data for marching as well as choosing the increment in velocity (or Mach number) rather than an increment Δz in physical space will avoid the difficulties stemming from mathematical ill-posedness in 3D marching. However, some successful computations with using Δz for traversing the small distance from the sonic surface down to the (resulting) surface data have been performed also.

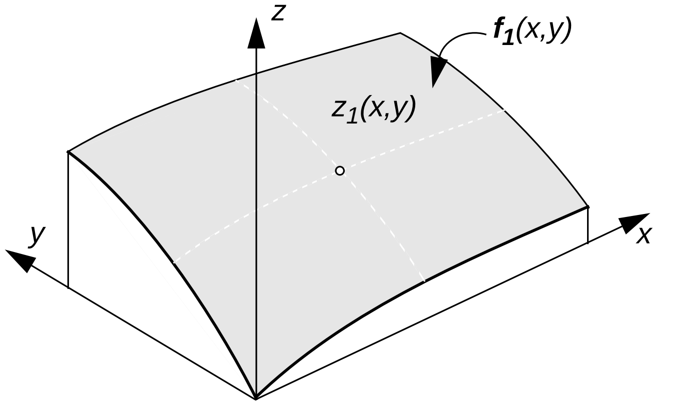

Figure 5: Surface with initial values distribution for cross-flow marching

For establishing an Euler accurate method of 3D marching we realize that we need the partial derivatives in 3D space of 5 variables of state: pressure, density and the 3 velocity components, represented here by a vector **f**, see equation (3a). We assume to have given boundary conditions along a given surface $z_1(x, y)$ where **f** is distributed according to $\mathbf{f}_1(x, y)$, see Fig. 5. We set up the steady 3D Euler equations in non-conservative form along this surface, i. e. continuity, 3 momentum equations and the energy equation, to obtain a system of 5 linear equations (3) for the 15 unknown gradients in 3D space. Analytic quality of z_1 and athe distribution $\mathbf{f}_1$ along z_1 yields partial derivatives which are related to the gradients according to (3b), thus providing the remaining 10 equations for solving the system (3). This locally linear system allows for establishing a numerical cross flow marching procedure equivalent to the inverse marching with the inverse MOC.

$$A_{ij} \cdot \mathbf{f}_{\mathbf{x}} = B_i; \qquad (3)$$

$$\mathbf{x} = (x, y, z), \qquad \mathbf{f} = (p, \rho, u, v, w), \qquad A_{ij} = A_{ij}(\mathbf{f}); \qquad (3a)$$

$$\frac{\partial \mathbf{f}_1}{\partial x} = \mathbf{f}_x + \mathbf{f}_z \cdot \frac{\partial z_1}{\partial x}, \qquad \frac{\partial \mathbf{f}_1}{\partial y} = \mathbf{f}_y + \mathbf{f}_z \cdot \frac{\partial z_1}{\partial y}; \qquad (3b)$$

4 Initial Boundary Value problems

With 2D, axisymmetric and finally fully 3D inverse marching in the local supersonic flow domains of our practical interest here available, we still ask for axisymmetric and fully 3D Euler accurate numerical procedures to provide the initial data for this inverse marching. Here we learn from the elliptic part of the hodograph design procedure briefly mentioned above.

4.1 Fictitious Gas Technique

The *fictitious* extension of the elliptic boundary value problem in the hodograph yielding data along the sonic locus can be traced back into the 5 Euler equations for 3D flow and shows an interesting physical interpretation, as depicted system (4):

$$\nabla\cdot(\rho \boldsymbol{q}) = 0,$$

$$\boldsymbol{q}\cdot\nabla\boldsymbol{q} + \nabla p/\rho = F, \qquad (4)$$

$$p/(\gamma-1)\rho + p/\rho + q^2/2 = H - \Gamma$$

We see the familiar Euler equations consisting of continuity equation, 3 momentum equation and one energy equation, with variables p for pressure, ρ for density and the velocity vector **q**(u, v, w); H is the total enthalpy, (the gas equation of state completing this system) . Allowing for a type change as in the hodograph, we arrive at additional terms at the right hand side of the momentum and energy equation: F can be interpreted as an imposed ‘gravity’ force field and Γ is an added energy distribution. This makes it clear that the initial work with this technique was on the level of the potential equation: A ‘*Fictitious Gas*’ model therefore was computed allowing the potential formulation. The variables F and Γ in (4) therefore have to be in a balance analog to potential and kinetic energy in classical mechanics in order to be conservative in the total potential, see [4].

Using the Euler equations, however, for such modelling does not require the potential balance in principle anymore: Setting F = 0 and distributing Γ as a function of the velocity difference to sonic flow conditions is an effective method to enforce an elliptic continuation with an Euler code for subsequent use of the MOC or 3D crossflow marching procedure. Physical interpretation of this model as a controlled energy (heat) removal and re-addition all within the local domain where $q > a^*$, that is where the velocity exceeds the critical speed of sound, is at least conceptually interesting: there have been some concepts proposed in other areas of fluid mechanics, using heat addition or cooling of the flow....

4.1.1 Supercritical wings

Applications of this design approach using the 2-step procedure to compute local flow domains with controlled quality have been obtained by adapting several numerical Euler codes to allow for such energy and momentum modifications.

Results for transonic (supercritical) airfoils and wings have been obtained by P. Li et al [5] showing increased aerodynamic efficiency because of wave drag reduction. The next step so far is an implementation in a time-accurate Navier Stokes CFD code by M. Trenker et al [6] to both design fully viscous (nearly) shock-free airfoil flows and study unsteady flow control occurring in the cyclic variations on helicopter rotor blades.

In the following, the method implemented in [5] will be illustrated for both transonic and supersonic applications. Fig. 6 shows an elliptic unswept wing in transonic flow with an originally strong recompression shock removed.

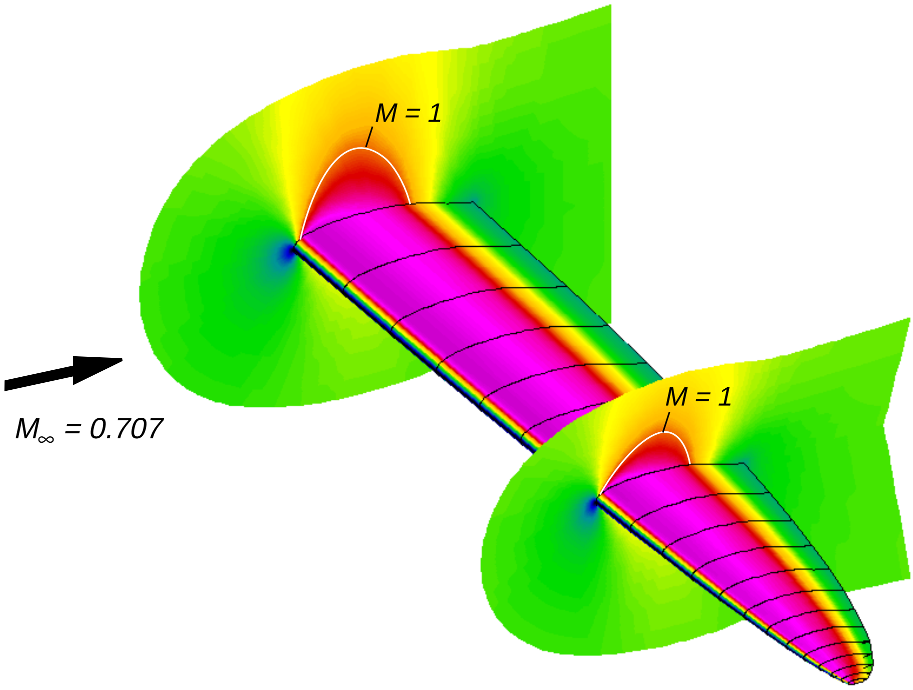

Figure 6: Euler CFD simulation of an elliptic wing in transonic flow with fictitious gas model within sonic surface: modifying the surface with initial conditions for shock-free redesign

4.1.2 Supersonic wings

Following the classical principle of obtaining the flow past a swept wing with infinite aspect ratio by superimposing 2D airfoil flow with a constant spanwise velocity normal to it suggests an extension of our design method for 3D wings to arrive at supersonic flows with reduced cross-flow shocks. Vector addition then simply requires increasing the threshold velocity where the modified energy equation is used, from sonic conditions to a supersonic Mach number. We know that an infinite swept wing with subsonic leading edges in supersonic flow will not have a detached bow shock: this has vanished upstream. 2D recompression shocks on the upper wing surface become cross flow shocks in supersonic flow. These are successfully removed for superimposition of a shock-free airfoil with a constant span-wise flow component.

A wing with finite aspect reatio, however, has a bow shock like the configuration Fig. 2a, and the ficti-tious gas technique might be used starting at a supersonic threshold velocity to favorably influence the flow field by suppressing cross flow shocks on the wing surface. This is reached, however, not with completely elliptic fictitious gas: flow conditions between Mach = 1 and the threshold mach number must behave hyperbolic and hence there is no guarantee that shocks can be romoved completely.

Fig. 7 shows a comparison of wing flows redesigned with the Euler code CFL3D extended to allow for F. G. extensions, both for transonic and supersonic operating conditions. A diagram shows an analysis of the fictitious gas law resulting from the modified energy equation in (4), the fat curves apply to the domains within the locally modified flow domains.

There is a relation between sweep and Mach number M_∞ for these two wing examples with high aspect ratio: The component M_N normal to the leading edge of the supersonic swept wing is just equal to the Mach number M_∞ of the unswept transonic wing, which results in approximately the same area of design modification for both wings on the upper surface.

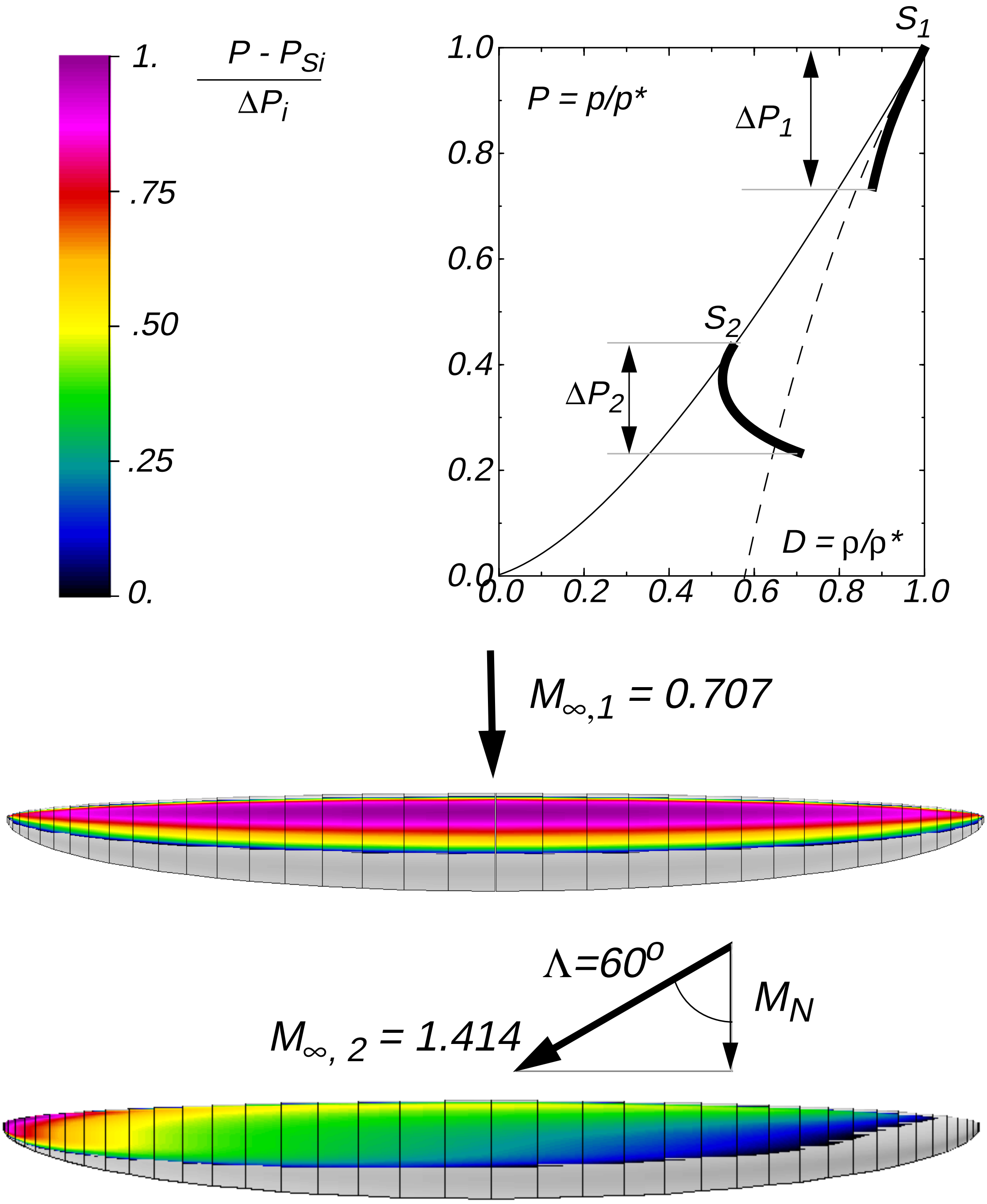

Figure 7: Comparison of transonic flow past unswept wing with supersonic flow past swept wing without crossflow shock: Fictitious gas method to remove recompression and cross flow shocks. Modified pressure - density relations within fictitious domains. Isofringes indicate areas of design modification.

4.2 Given Shock waves

Crossflow shocks on supersonic wings have been removed relatively easily, learning from the swept wing principle. Shock waves extending far into the flow field, as felt by the sonic boom of an aircraft flying with supersonic speeds, cannot be influenced by this approach. But bow waves of a limited extent into space occur with inlet diffuser flows where an oblique ramp shock ideally hits the inlet lip and will form there a relected system of shocks performing the needed compression of flow for the propulsion engine. We return, therefore, to the Euler method of characteristics and apply it to plane and axisymmetric flow fields with plane oblique, conical or curved shocks.
The MOC code written by Y. J. Qian has been used to shed light into some hitherto overseen details of the classical solution for cone flow in supersonic Mach numbers [7]. An example with a given curved axisymmetric bow shock in uniform supersonic upstream flow is depicted in Fig. 8. We see an extension of the charactsristic net of Mach waves beyond the body contour (an axisymmetric shaft ramp *resulting* from the computation, but physically *causing* the shock wave to occur in the flow), also showing a limit surface within the flow where the computed characteristics turn back off the axis this way providing a second branch of the solution. Being situated within the solid boundary, this solution may have academic value only, but it also tells us that marching in charactsristic variables (equivalent to the inverse marching direction in hodograph variables mentioned earlier) provides a complete solution, while a marching procedure in physical space coordinates would cause a breakdown of the code if the (generally unknown) limit surface is reached.

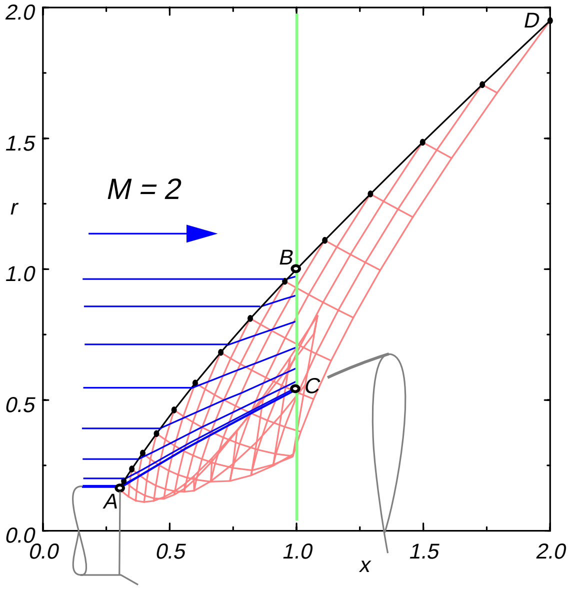

Figure 8: Result of inverse method of characteristics: axisymmetric ramp flow represented by given curved shock wave, stream lines and characteristics in meridional plane.

4.1.1 Waverider Design

Conical flows with oblique shocks have been used to design 3D waveriders in supersonic flow and with applications to hypersonic flight vehicles. The 3D marching procedure applied to start from a given arbitrary 3D curved oblique shock surface in uniform high supersonic speed flow has been used to arrive at 3D ramp surfaces generating such shocks [8], this way providing a 3D generalization of the waverider principle. K. Jones [9] has computed many examples of waveriders with various shapes, and made comparisons with Euler analysis using the designed shapes as input verifying the shock waves which have been used as design input before.

4.1.2 Osculating Axisymmetric Flows

Final examples mentioned here are 3D generalizations of the waverider design concept by using the axisymmetric MOC for a high order approximation to compute the local flow field between a shock wave and the ramp causing it: Assumption of a locally axisymmetric flow within soch flows allows to compose general waverider shapes with arbitrary shocks and vehicle planform shapes [10], see Fig. 9. We have termed this concept “Osculating Axisymmetry” - (OA) because of a very economic use of axisymmetric flow elements for design of quite general 3D body shapes.

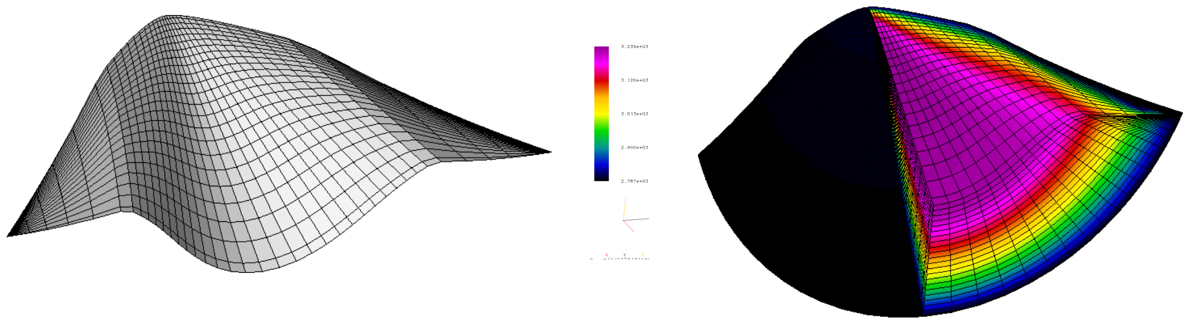

Figure 9: Supersonic waverider configuration computed from given shock wave, illustration of flow field pressure distribution between shock and surface.

5 Tools for Accelerated Optimization

So far, most of the examples shown here are rather academic studies to arrive at flows with some special property to be improved, like wave drag reduction of configurations in high speed flow.
As indicated,though, for the OA concept, which is strictly not an exact method compared to the other examples illustrated, we are interested to make use of theoretical concepts for practical and robust computational tools to be implemented in existing, reliable and fast CFD codes. Practical design goals in most cases include a suitable parameterization of surface geometries, to provide a link to necessary structural investigations, model and product definition and the related CAD modelling. For most practical cases we therefore need to extract knowledge from *true inverse* design to help making a toolbox for optimization with automated algorithms and perhaps still including some elements of inverse concepts by prescribing suitable target functions with a realistic physical background.
The above-mentioned swept wing in supersonic flow has been further refined by applying the kind of aerodynamic knowledge base as advocated here. After that, a suitable parameterization of the wing geometry including many variations but also the final design, was input for a numerical optimization effort [11] using a genetic algorithm: A costly test, but it confirmed our manually found result. From this we learn about the value of *true inverse* flow example construction.

6 Conclusion

These final remarks about tool improvement by suitable geometry parameterization ensure the value of the outlined inverse aerodynamic design methods in a time when iterative approaches to arrive at improved engineering solutions are the main route of design strategies now, made possible because of availability of high performance computers and efficient optimization software.
Beyond that, we may embed such concepts in ever-modernized software as comparatively small tool components to automatically include idealized case studies in the rich variety of candidate configurations for optimization.

References

Most of the cited and several related papers and illustrations can be downloaded from the URL: http://www.as.go.dlr.de/~helmut/

1. Sobieczky, H.: *Related Analytical, Anolog and Numerical Methods in Transonic Airfoil Design* AIAA paper 79-1556 (1980)
2. Sobieczky, H., Qian, Y.J.: *Extended Mapping and Characteristics Techniques for Inverse Aerodynamic Design*. Proc. Int. Conference on Inverse Design Concepts and Optimization in Engineering Sciences, (ICIDES III), Washington, D.C. (1991),
3. Sobieczky, H.: *Die Berechnung Lokaler Räumlicher Überschallfelder*. Zeitschr. Angew. Math. Mech. 58, pp. T331 - T333 (1978)
4. Li, P., Sobieczky, H.: *Computation of Fictitious Gas Flow with Euler Equations*. Acta Mechanica (Suppl.) 4: pp. 251-257 (1994)
5. Li, P., Sobieczky, H., Seebass, A. R.: *A New Design Method for Supersonic Transport*. AIAA 95-1819CP, (1995)
6. Trenker, M., Geissler, W., Sobieczky, H.: Airfoils with Dynamic Transonic Flow Control. AIAA 00-4419, (2000)
7. Sobieczky, H., Zores, B., Wang, Z., Qian, Y.J.: *High Speed Flow Design Using Osculating Axisymmetric Flows*. Proc. 3rd Pacific Int. Conf. on Aerospace Science and Technology (1997).
8. Sobieczky, H., Dougherty, F. C., Jones, K. D.: *Hypersonic Waverider Design from Given Shock Waves*. Proc. First Int. Hypersonic Waverider Symposium, University of Maryland, (1990)
9. Jones, K. D., Sobieczky, H., Seebass, A. R., Dougherty, F. C.: *Waverider Design for Generalized Shock Geometries*. J. Spacecraft and Rockets Vol. 32, No. 6 pp. 957 - 963 (1995)
10. Qian, Y., Sobieczky, H., Eggers, Th.: *Waverider Design with Parametric Flow Quality Control by Inverse Method of Characteristics*. In: M. Tanaka, G.S. Dulikravich, (Eds.), *Inverse Problems in Engineering Mechanics*, Proc. ISIP2000, Nagano, Japan (2000)
11. Klein, M., Sobieczky, H.: *Sensitivity of aerodynamic optimization to parameterized target functions*. In: M. Tanaka, G.S. Dulikravich, (Eds.), *Inverse Problems in Engineering Mechanics*, Proc. ISIP2001, Nagano, Japan (2001)